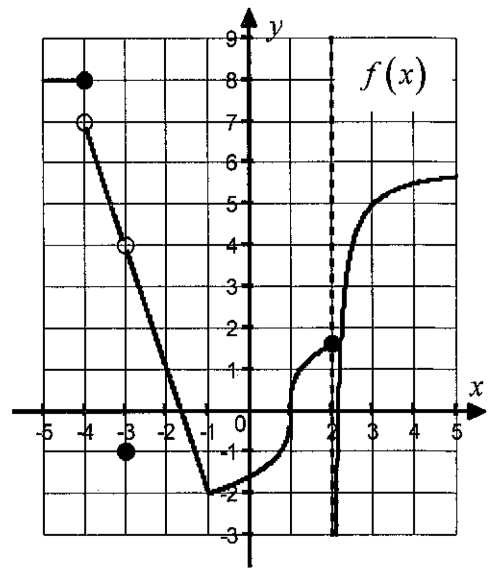
AP CALCULUS AB
Unit 1
Exam – Sample

Name Key
Date
Period

No calculators may be used on this portion of the exam.

1. Use the graph of f to evaluate or approximate (whichever is appropriate) each expression.



a. $\lim_{x\to-3} f(x) = 4$

b. $\lim_{x\to2^+} f(x) = -\infty$

c. $\lim_{x\to-1} f(x) = -2$

d. $f'(-4.5) = 0$

e. $f'(-2) = -3$

f. $f'(-1)$ undefined

g. $f'(0) \approx \frac{1}{2}$ (any value between $\frac{1}{2}$ and 1 is acceptable)

h. $f'(1)$ undefined

i. $\lim_{x\to3}\left(x^2 f(x)\right) = 9 \cdot 5 = \boxed{45}$

j. $\lim_{x\to1}\left(f(x) - 2x\right) = 0 - 2 = \boxed{-2}$

2. Use the *limit definition of the derivative* to find $g'(x)$ for the function $g(x) = 2 - \frac{1}{x}$.

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \frac{\cancel{2} - \frac{1}{x+h} - (\cancel{2} - \frac{1}{x})}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x} - \frac{1}{x+h}}{h} \cdot \frac{x(x+h)}{x(x+h)} = \lim_{h \to 0} \frac{\cancel{x} + \cancel{h} - \cancel{x}}{\cancel{h} x(x+h)}$$

$$= \frac{1}{x(x+0)} = \boxed{\frac{1}{x^2}}$$

3. a. Write a piecewise-defined function for $h(x) = \frac{25 - x^2}{|5 - x|}$.

$$h(x) = \begin{cases} \frac{(5+x)(5-x)}{5-x} & \text{if } x < 5 \\ \frac{(5+x)(5-x)}{-(5-x)} & \text{if } x > 5 \end{cases} = \boxed{\begin{cases} 5+x & \text{if } x < 5 \\ -(5+x) & \text{if } x > 5 \end{cases}}$$

b. Find $\lim_{x \to 5^-} h(x)$.

$$\lim_{x \to 5^-} h(x) = \lim_{x \to 5^-} (5+x) = \boxed{10}$$

4. Find each limit.

a. $\lim_{\theta \to \pi} \cos\left(\frac{1}{\theta - \pi}\right)$ $\cos\left(\frac{1}{0}\right)$ $\boxed{\text{DNE}}$

b. $\lim_{u \to 6} \frac{u - \sqrt{5u+6}}{u-6}$ $\frac{0}{0}$ $\cdot \frac{u + \sqrt{5u+6}}{u+\sqrt{5u+6}}$

$= \lim_{u \to 6} \frac{u^2 - 5u - 6}{(u-6)(u+\sqrt{5u+6})} = \lim_{u \to 6} \frac{\cancel{(u-6)}(u+1)}{\cancel{(u-6)}(u+\sqrt{5u+6})}$

$= \boxed{\frac{7}{12}}$

c. $\lim_{x \to 1} \tan^{-1}\left(\frac{3x-2}{x^2}\right) = \tan^{-1}(1)$

$= \boxed{\frac{\pi}{4}}$

d. $\lim_{z \to \infty} (z^5 - 2z^4) = \lim_{z \to \infty} z^4(z-2)$

$\infty - \infty$

$= \boxed{\infty}$

*e. $\lim_{x \to -\infty} \frac{3x^4 - 3x^2 + 1}{x - 1}$ $\frac{\infty}{-\infty}$

$= \lim_{x \to -\infty} \frac{\frac{3x^4}{x} - \frac{3x^2}{x} + \frac{1}{x}}{\frac{x}{x} - \frac{1}{x}}$

$= \lim_{x \to -\infty} \frac{3x^3 - 3x + \frac{1}{x}}{1 - \frac{1}{x}} = \boxed{-\infty}$

f. $\lim_{x \to 9^-} \frac{x^2 - 1}{9 - x} = \boxed{\infty}$ $\frac{80}{0}$

$\frac{80}{9 - 8.9} \to +$

g. $\lim_{u \to \infty} \frac{1-u}{\sqrt{3u^2 - u}} = \boxed{\frac{-1}{\sqrt{3}}}$

$\frac{-\infty}{\infty}$

*h. $\lim_{n \to \infty} \frac{3n^2 - 2}{n^2 - n^4} = \lim_{n \to \infty} \frac{\frac{3n^2}{n^4} - \frac{2}{n^4}}{\frac{n^2}{n^4} - \frac{n^4}{n^4}}$

$\frac{\infty}{-\infty}$

$= \lim_{n \to \infty} \frac{\frac{3}{n^2} - \frac{2}{n^4}}{\frac{1}{n^2} - 1} = \boxed{0}$

* For these $\frac{\infty}{\infty}$ cases, you may omit the work and use shortcuts. Keep in mind that this reduces your answer to a "right or wrong" answer.

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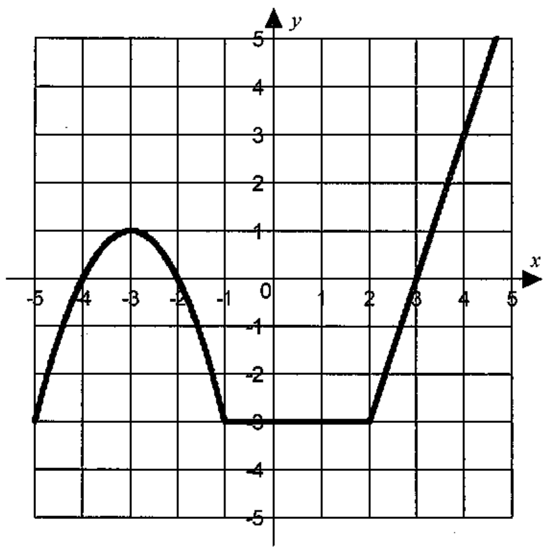
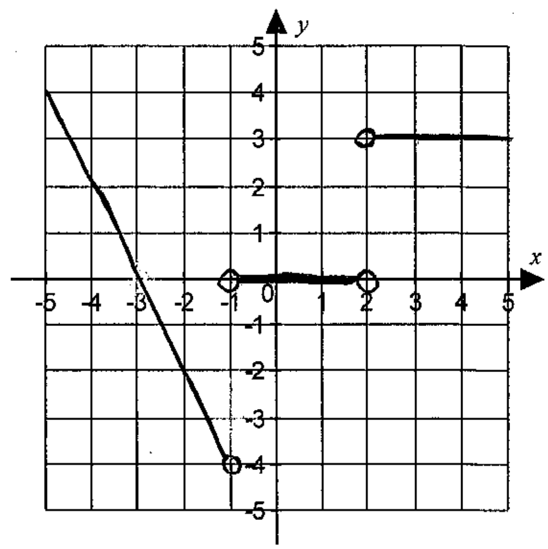
5. Using the graph of f given below, sketch a graph of f' on the axes provided.

$f(x)$



$f'(x)$



6. Suppose the function h, represented in the table below, is continuous for all real numbers.

x	-2	-1	0	1	2	3
$h(x)$	4	-1	-3	-1	2	3

a. Approximate $h'(-0.5)$.

$$h'(-0.5) \approx \frac{-3-(-1)}{0-(-1)} = \boxed{-2}$$

b. Suppose $g(x) = (h(x))^2 - 1$. Show that there must be a value c for $1 < c < 3$ such that $g(c) = 2$.

Since h is continuous, g is continuous on $[1, 3]$.

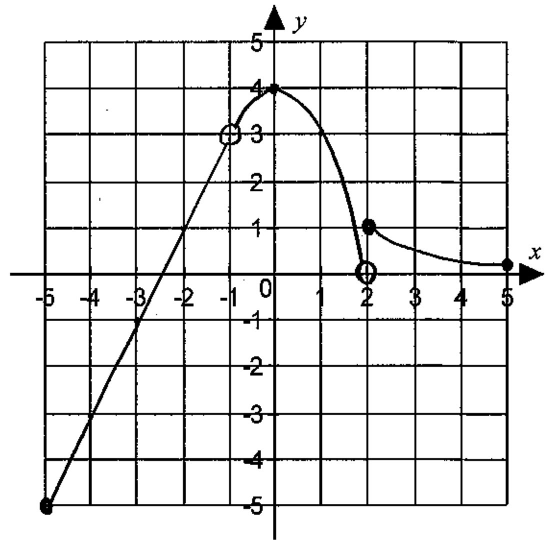
$g(1) = (h(1))^2 - 1 = 0 \longrightarrow g(1) < 2 < g(3)$

$g(3) = (h(3))^2 - 1 = 8$

By IVT, there exists some c on $(1, 3)$ such that $g(c) = 2$.

7. Let $g(x)=\begin{cases} 2x+5, & -5\le x<-1 \\ 4-x^2, & -1<x<2 \\ \dfrac{2}{x}, & 2\le x\le 5 \end{cases}$.

a. Sketch the graph of g using the axes given to the right.



b. Identify the value(s) of x such that g has a discontinuity. Classify each as either a *removable*, *jump*, or *infinite* discontinuity.

$x=-1$, removable

$x=2$, jump

c. Re-define g so that it is continuous at each removable discontinuity.

$g(-1)=3$

*8. Find the values of a and b that make $f(x)=\begin{cases} e^x+1, & x<0 \\ \cos x-2a, & 0\le x<\pi \\ b+6a, & x\ge\pi \end{cases}$ a continuous function.

$e^0+1=\cos(0)-2a$

$\boxed{a=-\frac{1}{2}}$

$\cos(\pi)-2a=b+6a$

$\cos(\pi)-2\cdot(-\frac{1}{2})=b+6\cdot(-\frac{1}{2})$

$\boxed{b=3}$

* This is the w/ calc section. Use your 89!! Setup & solve( ) command!!