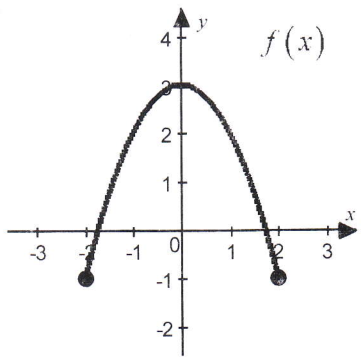
AP CALCULUS AB
Unit 4b
Exam – Sample

Name Key
Date
Period

No calculators may be used on this portion of the test.

1. The graphs of f and g are provided below on the closed interval $[-2, 2]$. Determine the coordinates of the absolute extreme value(s) for each function. If there is no absolute maximum and/or minimum, explain why.

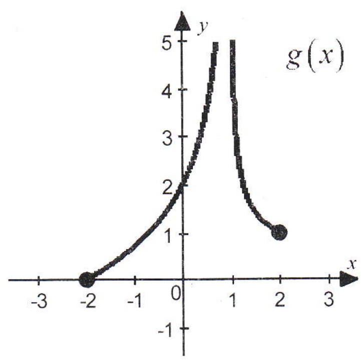
a.



Abs max: $(0, 3)$

Abs min: $(-2, -1)$, $(2, -1)$

b.



Abs max: none, since g has an infinite discontinuity at $x = 1$ such that $\lim_{x \to 1} g(x) = \infty$

Abs min: $(-2, 0)$

2. Wind caves are relatively small caves formed when wind erodes soft sandstone. In 2009, a geologist determined that the radius of a circular cave opening in the wall of a sandstone cliff was 1.4 meters. The same geologist recently returned to the sandstone cliff and determined that the radius of the circular cave opening is now 1.9 meters. Use *differentials* to estimate the change in area of the circular cave opening between the two measurements.

$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$dA = 2\pi r\, dr = 2\pi(1.4)(0.5) = \boxed{1.4\pi = \frac{7\pi}{5} \text{ m}^2}$

3. Suppose $f(1) = -5$, $f'(1) = 4$, and $f''(1) = 2$. Use *linearization* to approximate $f(0.75)$. Is the approximation an underestimate or overestimate? Justify your reasoning.

$f(x) \approx -5 + 4(x-1)$

$f(0.75) \approx -5 + 4(0.75-1) = -5 + 4(-0.25) = \boxed{-6}$

Since $f''(1) = 2 > 0$, the approximation is an underestimate

4. Find the absolute extreme value(s) for $g(x) = e^{3x-x^3}$ on the closed interval $[0, 2]$.

$g'(x) = e^{3x-x^3} \cdot (3-3x^2)$

$3 - 3x^2 = 0$

$x = \pm 1$

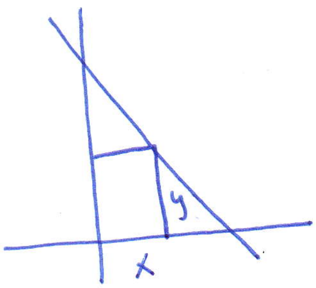
(omit $x=-1$, outside $[0,2]$)

x	$g(x)$
0	1
1	e^2
2	$\frac{1}{e^2}$

Abs max : $(1, e^2)$

Abs min : $(2, \frac{1}{e^2})$

5. Find the dimensions of the largest rectangle that can be inscribed within the region bounded by the line $y = 8 - 2x$ within the ~~second~~ first quadrant.



$A = xy$

$= x(8-2x)$

$= 8x - 2x^2$

Dimensions: 2×4

$A' = 8 - 4x = 0$

$x = 2$

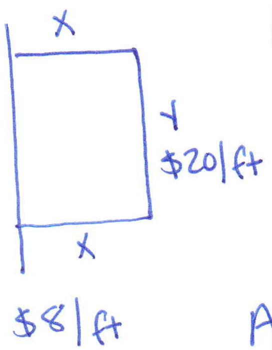
$A''(2) = -4 < 0 \rightarrow$ rel max

AP CALCULUS AB
Unit 4b
Exam – Sample

Name ______________________
Date ______________________
Period ______________________

Calculators may be used on this portion of the test.

6. An elementary school is building a fence around a rectangular playground. The playground is to border the school building and the side along the building does not need a fence. The side of the fence parallel to the building faces the road and will be built of wrought iron which cost 20 \$/ft while the 2 sides perpendicular to the building will be built of a cheaper material that cost 8 \$/ft. What is the largest area that can be fenced for a cost of \$ 4000?



$C = 8(2x) + 20y$

$4000 = 16x + 20y$

$y = \dfrac{1000 - 4x}{5}$

$A = xy$

$= x\left(\dfrac{1000-4x}{5}\right)$

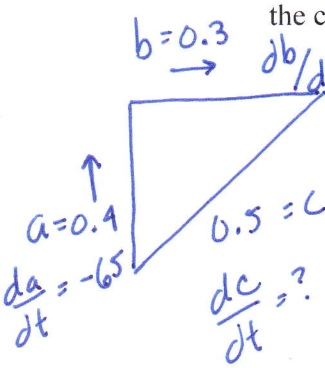
$A' = 200 - \dfrac{8x}{5} = \dfrac{1000 - 8x}{5} = 0$

$x = 125$

$A''(125) = -\dfrac{8}{5} < 0 \rightarrow$ max

$A(125) = \boxed{12500 \text{ ft}^2}$

7. An ambulance is racing north toward an intersection at 65 mph. A passenger car is traveling east away from the intersection at 40 mph. Find the rate of change of the distance between the ambulance and the passenger car when the ambulance is 0.4 miles from the intersection and the car is 0.3 miles from the intersection.



$a\dfrac{da}{dt} + b\dfrac{db}{dt} = c\dfrac{dc}{dt}$

$\dfrac{(0.4)(-65) + (0.3)(40)}{0.5} = \dfrac{dc}{dt} = \boxed{-28 \text{ mph}}$

8. Use *linearization* to approximate $\sqrt{16.443}$. Be sure to show all work that leads to your answer. Solutions with no work will receive NO CREDIT.

Let $f(x)=\sqrt{x}$ and $a=16$

$f(16)=4$

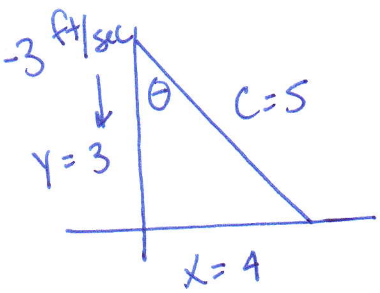
$f'(16)=\frac{1}{2\sqrt{16}}=\frac{1}{8}$

$f(x)\approx 4+\frac{1}{8}(x-16)$

$\sqrt{16.443}=f(16.443)\approx 4+\frac{1}{8}(16.443)-16$

$\boxed{\sqrt{16.443}\approx 4.055375}$

9. A 5 foot ladder is leaning against a house when its base starts to slide away. At what rate is the angle between the ladder and the wall changing when the base of the ladder is 4 feet from the wall if the ladder is sliding down the wall at a rate of 3 feet per second?



$\sin\theta=\frac{4}{5}$

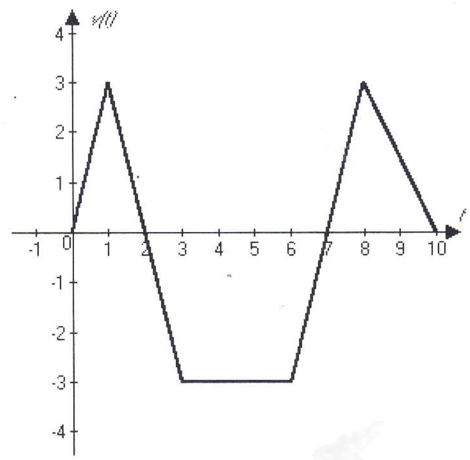
$\cos\theta=\frac{y}{c}$

$-\sin\theta\,\frac{d\theta}{dt}=\frac{1}{c}\,\frac{dy}{dt}$

$-\frac{4}{5}\,\frac{d\theta}{dt}=\frac{1}{5}\cdot(-3)$

$\boxed{\frac{d\theta}{dt}=\frac{3}{4}\,\frac{\text{rad}}{\text{sec}}}$

10. The accompanying figure shows the velocity (in m/s) of a body moving along a line from $t = 0$ seconds to $t = 10$ seconds. Answer the following.



a) What is the average acceleration over the first 5 seconds?

$a_{avg} = \frac{v(5) - v(0)}{5-0} = \frac{-3-0}{5-0} = \boxed{\frac{-3}{5}\ m/s^2}$

b) When is the body at rest?

$v(t) = 0$

$t = 0, 2, 7, 10$ sec

c) When is the body speeding up?

v: 0 + 2 − 7 + 10

a: 0 + 1 − 3 0 6 + 8 − 10

$(0,1) \cup (2,3) \cup (7,8)$

11. The position function for an object moving along a horizontal path is given by $x(t) = 10 - t^2 + t^4$, $t \geq 0$, where x is measured in feet and t is measured in minutes. Answer the following.

$v(t) = -2t + 4t^3$

$a(t) = -2 + 12t^2$

a) What is the acceleration at $t = 7$ minutes?

$a(7) = 586\ ft/min^2$

b) When is the body moving forward?

v: 0 − $\sqrt{2}/2$ +

$v(t) = -2t + 4t^3 = 0$

$t = 0, \sqrt{2}/2$

forward $(\sqrt{2}/2, \infty)$

c) What is the velocity when the acceleration is zero?

$a(t) = 0$

$t = \frac{\sqrt{6}}{6}$

$v(\sqrt{6}/6) = \frac{-2\sqrt{6}}{9}$ or $-0.544\ \frac{ft}{min}$