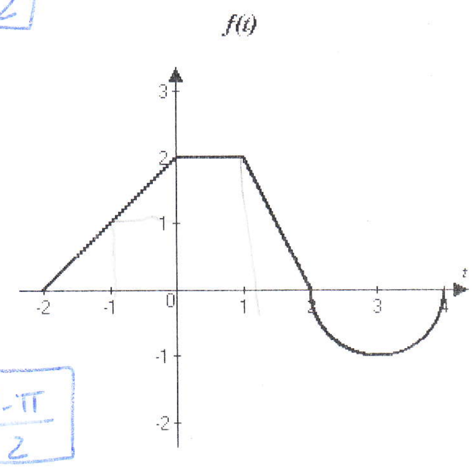
AP CALCULUS (AB)
Unit 5
Sample Test

Name ____________________
Date ____________________
Period ____________________

No calculators may be used on this portion of the test.

Suppose $g(x)=\int_{-1}^{x} f(t)\,dt$ and $h(x)=\int_{0}^{x^2} f(t)\,dt$. Use the graph of f shown to evaluate the following.



1. $g(-2)$ $= -\int_{-2}^{-1} f(t)\,dt = -\frac{1}{2}(1)(1) = \boxed{-\frac{1}{2}}$

2. $g(-1)$ $= \int_{-1}^{-1} f(t)\,dt = \boxed{0}$

3. $g(4)$ $= \int_{-1}^{4} f(t)\,dt = \frac{3}{2} + 2 + 1 - \frac{\pi}{2} = \boxed{\frac{9-\pi}{2}}$

4. $g'(1)$ $= f(1) = \boxed{2}$

5. $g''(0.5)$ $= f'(0.5) = \boxed{0}$

6. $h(-1)$ $= \int_{0}^{1} f(t)\,dt = \boxed{2}$

7. $h'\left(\sqrt{3}\right)$

$h'(x) = f(x^2)\cdot 2x$

$h'(\sqrt{3}) = f(3)\cdot 2\sqrt{3}$

$= \boxed{-2\sqrt{3}}$

Evaluate.

8. $\int \sec^2(2-x)\,dx$ $u=2-x$ $du=-dx$

$= \boxed{-\tan(2-x)+C}$

9. $\int e^x\left(1+e^x\right)^{10} dx$ $u=1+e^x$ $du=e^x dx$

$= \boxed{\frac{(1+e^x)^{11}}{11}+C}$

10. $\int \alpha \tan\alpha^2 \, d\alpha$

$=\int \alpha \frac{\sin\alpha^2}{\cos\alpha^2}\,d\alpha$

$u=\cos\alpha^2$

$du=-2\alpha\sin\alpha^2\,d\alpha$

$-\frac{1}{2}\int \frac{1}{u}\,du$

$=\boxed{-\frac{1}{2}\ln\left|\cos\alpha^2\right|+C}$

11. $\int \frac{z \; {}^{+3-3}}{3+z}\,dz$

$=\int\left(1-\frac{3}{3+z}\right)dz$

$= \boxed{z-3\ln|3+z|+C}$

Evaluate.

12. $\int_1^2 \left(u - 2^u\right) du$

$$= \frac{u^2}{2} - \frac{2^u}{\ln 2} \Big|_1^2$$

$$= \left(2 - \frac{4}{\ln 2}\right) - \left(\frac{1}{2} - \frac{2}{\ln 2}\right)$$

$$= \boxed{\frac{3}{2} - \frac{2}{\ln 2}}$$

or

$$\boxed{\frac{3\ln 2 - 4}{2\ln 2}}$$

13. $\int_0^3 |x^2 - 4| \, dx$

$x^2 - 4 = 0$

$x = \pm 2$

$-$ $+$

0 2 3

$$= -\int_0^2 (x^2-4)\,dx + \int_2^3 (x^2-4)\,dx$$

$$= -\left[\frac{x^3}{3} - 4x\right]_0^2 + \left[\frac{x^3}{3} - 4x\right]_2^3$$

$$= -\left(\frac{8}{3} - 8\right) + (9-12) - \left(\frac{8}{3} - 8\right)$$

$$= -\frac{16}{3} + 16 - 3$$

$$= \frac{39}{3} - \frac{16}{3}$$

$$= \boxed{\frac{23}{3}}$$

14. $\int_0^{1/2} \frac{1}{1+4y^2}\, dy$

$u = 2y$ $\quad u(0) = 0$

$du = 2\, dy$ $\quad u(\frac{1}{2}) = 1$

$$= \frac{1}{2}\int_0^1 \frac{1}{1+u^2}\, du$$

$$= \frac{1}{2}\tan^{-1}(u) \Big|_0^1$$

$$= \frac{1}{2}\left(\tan^{-1} 1 - \tan^{-1} 0\right)$$

$$= \frac{1}{2}\left(\frac{\pi}{4} - 0\right)$$

$$= \boxed{\frac{\pi}{8}}$$

15. $\int_0^1 \frac{x\, dx}{\sqrt{1-x^2}}$

$u = 1 - x^2$ $\quad u(0) = 1$

$du = -2x\, dx$ $\quad u(1) = 0$

$$= -\frac{1}{2}\int_1^0 u^{-1/2}\, du$$

$$= \sqrt{u}\,\Big|_0^1$$

$$= \boxed{1}$$

AP CALCULUS (AB)
Unit 5
Sample Test

Name ______________________
Date ______________________
Period ______________________

Calculators may be used on this portion of the test.

A group of teachers attends a diet support group every week. A teacher monitors her rate of weight loss, $W(t)$, in pounds per week. Use the data from the table to answer the following.

t	$W(t)$
0	4.0
2	3.1
5	2.8
6	2.3
7	1.7
9	1.4
10	1.9

16. Using units, describe what $\int_0^{10} W(t)\,dt$ represents.

$\int_0^{10} w(t)\,dt$ represents the total loss of weight of the teacher, in pounds, from $t=0$ to $t=10$ weeks.

17. Use trapezoids with three subdivisions to approximate $\int_0^{10} W(t)\,dt$.

$$\int_0^{10} w(t)\,dt \approx \tfrac{1}{2}(5)(4+2.8)+\tfrac{1}{2}(2)(2.8+1.7)+\tfrac{1}{2}(3)(1.7+1.9)$$
$$= \boxed{26.9 \text{ pounds}}$$

18. When the teacher first joined the diet support group, her weight was 184 pounds. Estimate her weight at week 10 using three rectangles with right endpoints as sample points. (Hint: $W(t)$ is the rate of weight *loss*.)

$$\int_0^{10} w(t)\,dt = \text{Weight at } t=10 - \text{Weight at } t=0$$
$$\text{Weight at } t=10 = 184 - \int_0^{10} w(t)\,dt$$
$$\approx 184 - (5\cdot 2.8 + 2\cdot 1.7 + 3\cdot 1.9)$$
$$= \boxed{160.9 \text{ lbs}}$$

Suppose a particle travels along a linear path at a velocity of $v(t) = \frac{1}{\sqrt{t}} - \frac{3}{2}$, in meters per second, $t \geq 1$ second.

19. Calculate $\int_1^4 |v(t)|\,dt$.

$\int_1^4 |v(t)|\,dt = -\int_1^4 v(t)\,dt = \frac{5}{2}$

20. Explain what $\int_1^4 |v(t)|\,dt$ represents. Include units in your explanation.

$\int_1^4 |v(t)|\,dt$ represents the total distance traveled, in meters, over the time interval $1 \leq t \leq 4$, in seconds.

21. Calculate the displacement of the particle over the interval $[1, 6]$.

$\int_1^6 v(t)\,dt = 2\sqrt{6} - \frac{19}{2} = \boxed{-4.601 \text{ m}}$