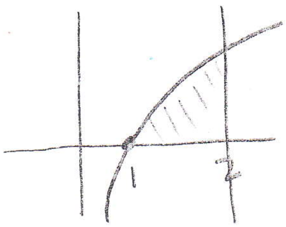
AP CALCULUS AB
Unit 6
Exam – Sample

Name ____________________
Date ____________________
Period ____________________

No calculators may be used on this portion of the test.

1. The base of a solid is the region bounded by the curves $y = \ln x$, $x = 2$, and the x-axis. The cross sections perpendicular to the x-axis are semicircles with diameters running from the x-axis to the curve. Set up an integral expression to find the volume of the solid. **Do not solve**.

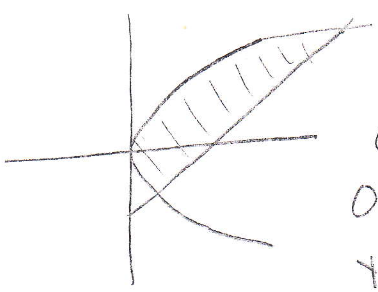


$$\boxed{V = \frac{\pi}{8}\int_1^2 (\ln x)^2\,dx}$$

or $V = \frac{\pi}{2}\int_1^2 \left(\frac{\ln x}{2}\right)^2 dx$

$6y = x - 9$
$y = \frac{x}{6} - \frac{3}{2}$

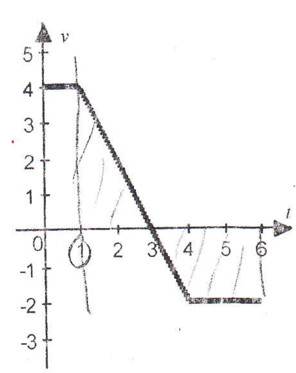
2. Set up an integral expression to find the area bounded by the curves $x = 6y + 9$ and $x = 3y^2$. **Do not solve**.



$6y + 9 = 3y^2$
$0 = 3y^2 - 6y - 9$
$0 = 3(y^2 - 2y - 3)$
$0 = 3(y-3)(y+1)$
$y = 3, -1$

$$\boxed{A = \int_{-1}^{3} (6y + 9 - 3y^2)\,dy}$$

3. The graph of the velocity function of a particle moving along a linear path is given to the right, where velocity is measured in m/s. Determine the average velocity of the particle over the interval $[1, 6]$.



$V_{avg} = \frac{1}{6-1}\int_1^6 v(t)\,dt$

$= \frac{1}{5}\left[\frac{1}{2}(2)(4) - \frac{1}{2}(1)(2) - (2)(2)\right]$

$= \frac{1}{5}(4 - 1 - 4)$

$= \boxed{\frac{-1}{5} \text{ m/s}}$

4. Find the general solution for each of the following differential equations.

a. $\frac{dy}{dx} = \frac{\cos(2x)}{y^4}$

$\int y^4\,dy = \int \cos(2x)\,dx$

$u = 2x$

$du = 2\,dx$

$\frac{y^5}{5} = \frac{1}{2}\int \cos u\,du$

$y^5 = \frac{5}{2}\sin(2x) + C$

$\boxed{y = \sqrt[5]{\frac{5}{2}\sin(2x) + C}}$

b. $x^2y^2 + y' = 4y^2$

$x^2y^2 + \frac{dy}{dx} = 4y^2$

$\frac{dy}{dx} = y^2(4 - x^2)$

$\int \frac{dy}{y^2} = \int (4 - x^2)\,dx$

$-\frac{1}{y} = 4x - \frac{x^3}{3} + C_1$

$-\frac{1}{y} = \frac{12x - x^3 + C_2}{3}$ $\langle C_2 = 3C_1 \rangle$

$\boxed{y = \frac{-3}{12x - x^3 + C}}$

5. Find the particular solution for each of the following differential equations with given initial conditions.

a. $\csc\theta \frac{dr}{d\theta} = 2\sqrt{r}$; $r(\pi) = 16$

$\int \frac{dr}{2\sqrt{r}} = \int \sin\theta\,d\theta$

$\frac{1}{\cancel{2}}\cancel{2}\, r^{1/2} = -\cos\theta + C$

$r(\pi) = 16$ $\quad 16^{1/2} = -\cos\pi + C$

$4 = -(-1) + C$

$4 = 1 + C$

$C = 3$

$\sqrt{r} = -\cos\theta + 3$

$\boxed{r = (3 - \cos\theta)^2}$

b. $\frac{y'}{2x-1} = y + 4$; $y(1) = -1$

$\frac{dy}{dx} = (y+4)(2x-1)$

$\int \frac{dy}{y+4} = \int (2x-1)\,dx$

$\ln|y+4| = x^2 - x + C$

$|y+4| = Ce^{x^2-x}$

$y = Ce^{x^2-x} - 4$

$-1 = Ce^{0} - 4$

$C = 3$

$\boxed{y = 3e^{x^2-x} - 4}$

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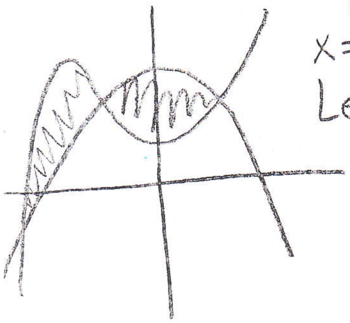
6. Find the area of the region bounded by the curves $h(x)=5-x^2$ and $r(x)=x^3+2x^2-2x+2$.



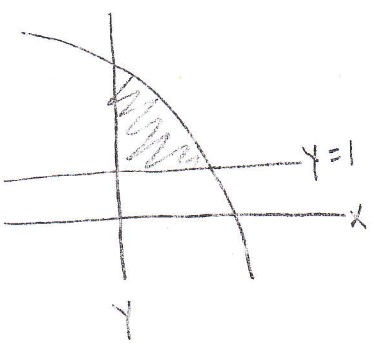
$h(x)=r(x)$

$x=-3.33005,\ -0.798360,\ 1.128419$

Let $a=-3.33005$, $b=-0.798360$, $C=1.128419$

$$A=\int_a^b (r(x)-h(x))\,dx+\int_b^c (h(x)-r(x))\,dx=\boxed{12.801219}$$

7. A region is bounded by the curves $y=6-e^x$, $y=1$, and $x=0$. Find the volume generated by revolving the region about the y-axis.



$y(0)=6-e^0=5$

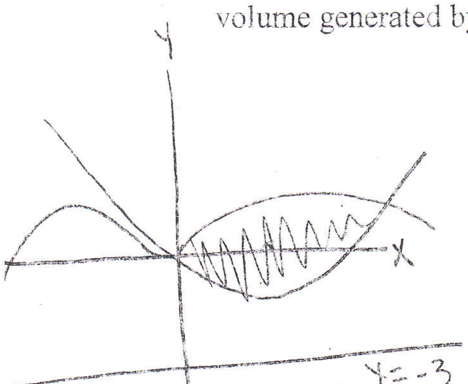
$y=6-e^x$

$x=\ln(6-y)$

$$V=\pi\int_1^5 (\ln(6-y))^2\,dy$$

$$=\boxed{15.258944}$$

8. Consider the region bounded by the curves $f(x)=x^2-x$ and $g(x)=\sin(x^2)$. Find the volume generated by revolving the region about the line $y=-3$.



$f(x)=g(x)$

$x=0,\ 1.507159$

let $a=1.507159$

$$V=\pi\int_0^a \left[(g(x)+3)^2-(f(x)+3)^2\right]dx$$

$$=\boxed{16.221812}$$

9. The population of Kennesaw, Georgia was 11,500 in the year 1995, and 30,000 in the year 2005. If the population growth is proportional to the population, find an equation that represents the population at time t and use it to predict the population in 2015. **Round your final answer to the nearest whole number.**

$t=0 \rightarrow 1995$

$30{,}000 = 11{,}500\, e^{K \cdot 10}$

$y = Ce^{Kt}$

$K = -\frac{1}{10} \ln\left(\frac{23}{60}\right) = 0.095885$

$C = 11{,}500$

$y(20) = 11{,}500\, e^{20K} = 78{,}260.869565$

$\boxed{78{,}261 \text{ people}}$

10. On a hot summer day, a bottle of water has been left outside in the heat. The temperature of the water is determined to be $90\,°F$. The bottle of water is placed in a refrigerator set to $35\,°F$. In 60 minutes, the water has cooled to $73\,°F$. How long will it take for the temperature of the water to reach $40\,°F$? (Hint: Use Newton's Law of Cooling.)

$T = Ce^{Kt} + T_e$

$73 = 55\, e^{60K} + 35$

$T = Ce^{Kt} + 35$

$K = \frac{1}{60} \ln\left(\frac{38}{55}\right) = -0.00616245$

$90 = Ce^{0} + 35$

$C = 55$

$40 = 55e^{Kt} + 35$

$\boxed{t = 389.113925 \text{ min}}$

$T = 55e^{Kt} + 35$