**AP CALCULUS AB**
**Unit 2**
**Exam – Sample**

Name Key
Date
Period

No calculators may be used on this portion of the test.

1. Consider the curve $C$ defined by $x^2 + xy - y^2 = 5$.

   a. Calculate $\frac{dy}{dx}$.

$$2x + x\frac{dy}{dx} + y - 2y\frac{dy}{dx} = 0$$

$$2x+y = 2y\frac{dy}{dx} - x\frac{dy}{dx}$$

$$2x+y = \frac{dy}{dx}(2y-x)$$

$$\boxed{\frac{dy}{dx} = \frac{2x+y}{2y-x}}$$

Note: $\frac{dy}{dx} = \frac{-2x-y}{x-2y}$ is ok, too!!

   b. Find an equation for the line tangent to $C$ at the point $(3, -1)$.

$$\left.\frac{dy}{dx}\right|_{(3,-1)} = \frac{6-1}{-2-3} = \frac{5}{-5} = -1$$

$$\boxed{y+1 = -(x-3)}$$

2. Calculate $\left.\frac{d^3y}{dx^3}\right|_{x=4}$ given $y = \frac{2}{\sqrt{x}}$. $= 2x^{-1/2}$

$32$
$16$
$192$
$32$
$512$

$$\frac{dy}{dx} = -x^{-3/2}$$

$$\frac{d^2y}{dx^2} = \frac{3}{2}x^{-5/2}$$

$$\frac{d^3y}{dx^3} = -\frac{15}{4}x^{-7/2}$$

$$\left.\frac{d^3y}{dx^3}\right|_{x=4} = -\frac{15}{4\cdot 4^{7/2}} = -\frac{15}{2^2\cdot 2^7}$$

$$= -\frac{15}{2^4\cdot 2^5} = -\frac{15}{16\cdot 32} = \boxed{-\frac{15}{512}}$$

3. A normal line is the line perpendicular to a given tangent line that passes through the point of tangency. Given $f(4)=3$ and $f'(4)=2$, find the equations for both the tangent and normal lines to $f(x)$ at $x=4$.

Tangent: $y-3=2(x-4)$

Normal: $y-3=-\frac{1}{2}(x-4)$

4. At what point(s) on the curve $y=(\ln(x+4))^2$ is the tangent horizontal?

$$y' = 2\ln(x+4)\cdot\frac{1}{x+4} = \frac{2\ln(x+4)}{x+4}$$

$$y'=0 \longrightarrow 2\ln(x+4)=0$$

$$e^{\ln(x+4)} = e^{0}$$

$$x+4=1$$

$$x=-3$$

$$\boxed{(-3,0)}$$

$$y(-3)=(\ln(-3+4))^2=(\ln(1))^2=0$$

Note: Work is not shown. A hint is given for each in ( ).

5. Find the derivative for each of the following functions. (Don't panic!! There will not be this many derivative problems on the exam. We just want you to practice a lot!!)

a. $z=\frac{e^y}{y^2}$ (Quotient Rule)

$$\boxed{z' = \frac{e^y(y-2)}{y^3}}$$

b. $y=\frac{x^2-2\sqrt{x}}{x}$ (Simplify & Power Rule)

$$\boxed{y' = 1 + \frac{1}{x^{3/2}}}$$

c. $R(t)=\frac{5}{t^{3/5}}$ (Power Rule)

$$\boxed{R'(t) = -\frac{3}{t^{8/5}}}$$

d. $h(r)=(r^2-2r)e^r$ (Product Rule)

$$\boxed{h'(r) = e^r(r^2-2)}$$

e. $r=\frac{e^u}{\cos u}$ (Quotient Rule)

$$\boxed{r' = \frac{e^u(\cos u+\sin u)}{\cos^2 u}}$$

f. $f(\theta)=\frac{1}{\tan^2\theta}$ (Chain Rule)

$$\boxed{f'(\theta) = -\frac{2\sec^2\theta}{\tan^3\theta}} \text{ OR } \boxed{f'(\theta) = -2\csc^2\theta\cot\theta}$$

g. $y=\log 5x-5^x$ (Chain Rule)

$$\boxed{y' = \frac{1}{x\ln(10)} - 5^x\ln(5)}$$

h. $r(x)=\tan^{-1}(1-x)$ (Chain Rule)

$$\boxed{r'(x) = -\frac{1}{x^2-2x+2}}$$

i. $h(t)=\sqrt{2t}-\arcsin t$ (Chain Rule)

$$\boxed{h'(t) = \frac{1}{\sqrt{2t}} - \frac{1}{\sqrt{1-t^2}}}$$

j. $y=\frac{1}{\sqrt{e^x-x^2}}$ (Chain Rule)

$$\boxed{y' = \frac{2x-e^x}{2(e^x-x^2)^{3/2}}}$$

k. $y=x^{\ln x}$ (Logarithmic Differentiation)

$$\boxed{\frac{dy}{dx} = \frac{2x^{\ln(x)}\ln(x)}{x}}$$

l. $f(x)=(x-1)^4(2x+5)^5$ (Product & Chain Rules)

$$\boxed{f'(x) = 2(x-1)^3(2x+5)^4(9x+5)}$$

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6. Use the table below to answer the following. Show all work that leads to your answers.

| $x$ | $f(x)$ | $g(x)$ | $f'(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 2 | −2 | 3 | 5 | −8 |
| 4 | 3 | −4 | −1 | 3 |
| 6 | −5 | 3 | 2 | 0 |
| 8 | 2 | −3 | −4 | −8 |
| 10 | 2 | −5 | 2 | 1 |

a. Find $y'(4)$ given $y = x + g\left(\sqrt{x}\right)$.

$$y'(x) = 1 + g'(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

$$y'(4) = 1 + \frac{g'(2)}{2 \cdot 2} = 1 - \frac{8}{4} = \boxed{-1}$$

b. Find $y'(8)$ given $y = f(x)(g(x))^2$.

$$y'(x) = f(x) \cdot 2g(x)g'(x) + (g(x))^2 f'(x)$$

$$y'(8) = (2)(2)(-3)(-8) + (-3)^2(-4) = 96 - 36 = \boxed{60}$$

c. Find $f^{-1}(-5)$ given $y = f^{-1}(x)$.

$$y'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$y'(-5) = \frac{1}{f'(f^{-1}(-5))} = \frac{1}{f'(6)} = \boxed{\frac{1}{2}}$$

7. Suppose $g(x) = f^{-1}(x)$ and $f(x) = e^{-3x} - x$. Evaluate $g'(3)$.

$$g'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{f'(-0.327652)} = \boxed{-0.110901}$$

$$f'(x) = -3e^{-3x} - 1$$

$$f(x) = 3$$

$$x = -0.327652 = f^{-1}(3)$$

8. Find the points for any horizontal and vertical tangent lines to the curve defined by $y^3 - x^3 = 8y + 9x - 1$.

given $\frac{dy}{dx} = \frac{3x^2+9}{3y^2-8}$

Horizontal: $\frac{dy}{dx} = 0 \rightarrow 3x^2 + 9 = 0$

No real solutions

$\boxed{\text{No points of horizontal tangency}}$

Vertical: $\frac{dy}{dx}$ undef $\rightarrow 3y^2 - 8 = 0$

$$y = \pm \frac{2\sqrt{6}}{3} \approx \pm 1.632993$$

$\left(\frac{2\sqrt{6}}{3}\right)^3 - x^3 = 8\left(\frac{2\sqrt{6}}{3}\right) + 9x - 1$ $\qquad$ $\left(-\frac{2\sqrt{6}}{3}\right)^3 - x^3 = 8\left(-\frac{2\sqrt{6}}{3}\right) + 9x - 1$

$x = -0.799752$ $\qquad$ $x = 0.975627$

Points of Vertical Tangency: $(-0.799752, 1.632993)$, $(0.975627, -1.632993)$