AP CALCULUS AB
Unit 3
Exam – Sample

Name Key
Date ______
Period ______

No calculators may be used on this test.

1. Find each limit. Show all work that leads to your final answers.

a. $$\lim_{x\to 0}\frac{\overset{0}{x^2}}{\underset{0}{xe^x-x}}$$

$$=\lim_{x\to 0}\frac{\overset{0}{2x}}{\underset{0}{xe^x+e^x-1}}$$

$$=\lim_{x\to 0}\frac{2}{xe^x+e^x+e^x}$$

$$=\boxed{1}$$

b. $$\lim_{x\to\infty}\frac{\overset{\infty}{\ln x}}{\underset{\infty}{\sqrt{x}}}=\lim_{x\to\infty}\frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}}$$

$$=\lim_{x\to\infty}\left(\frac{1}{x}\right)\left(2\sqrt{x}\right)$$

$$=\lim_{x\to\infty}\frac{2}{\sqrt{x}}$$

$$=\boxed{0}$$

c. $$\lim_{x\to 1^+}\left(\overset{\infty}{\frac{1}{\ln x}}-\overset{-\infty}{\frac{1}{x-1}}\right)$$

$$=\lim_{x\to 1^+}\frac{x-1-\ln x}{x\ln x-\ln x}$$

$$=\lim_{x\to 1^+}\frac{1-\frac{1}{x}}{x\cdot\frac{1}{x}+\ln x-\frac{1}{x}}\quad\frac{0}{0}$$

$$=\lim_{x\to 1^+}\frac{\frac{1}{x^2}}{\frac{1}{x}+\frac{1}{x^2}}$$

$$=\boxed{\frac{1}{2}}$$

d. $$\lim_{x\to 0^+}(\cos x)^{\frac{1}{x^2}}\quad 1^\infty$$

Let $y=(\cos x)^{\frac{1}{x^2}}$

$$\lim_{x\to 0^+}\ln y=\lim_{x\to 0^+}\frac{\ln(\cos x)}{x^2}$$

$$=\lim_{x\to 0^+}\frac{\frac{1}{\cos x}\cdot(-\sin x)}{2x}\quad\frac{0}{0}$$

$$=\lim_{x\to 0^+}\frac{-\tan x}{2x}$$

$$=\lim_{x\to 0^+}\frac{-\sec^2 x}{2}$$

$$\ln\left(\lim_{x\to 0^+}y\right)=-\frac{1}{2}$$

$$\lim_{x\to 0^+}(\cos x)^{\frac{1}{x^2}}=\boxed{e^{-\frac{1}{2}}}$$

2. Consider the function f defined below with the following derivatives.

$$f(x)=\frac{2x^2-8}{(x-4)^2} \qquad f'(x)=\frac{-16(x-1)}{(x-4)^3} \qquad f''(x)=\frac{16(2x+1)}{(x-4)^4}$$

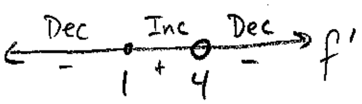
a. Identify intervals of increase/decrease and concavity in f.

$f' = 0$

$x = 1$

f' undef

$x = 4$

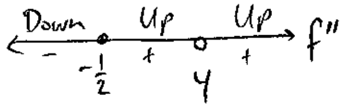


Increase: $(1, 4)$

Decrease: $(-\infty, 1) \cup (4, \infty)$

$f'' = 0$

$x = -\frac{1}{2}$



f'' undef

$x = 4$

Concave Down: $(-\infty, -\frac{1}{2})$

Concave Up: $(-\frac{1}{2}, 4) \cup (4, \infty)$

b. Identify all relative extreme values and points of inflection in f.

Relative Min: $(1, -\frac{2}{3})$

Relative Max: None

$f(1) = \frac{-6}{9}$

POI: $(-\frac{1}{2}, -\frac{10}{27})$

$$f(-\tfrac{1}{2}) = \frac{\frac{1}{2}-8}{(-\frac{9}{2})^2} = \frac{\frac{1}{2}-8}{\frac{81}{4}} \cdot \frac{4}{4} = \frac{2-32}{81} = -\frac{30}{81}$$

c. Sketch f in the window provided. Label all asymptotes and intercepts.

Intercepts: $2x^2 - 8 = 0$

$x = \pm 2$

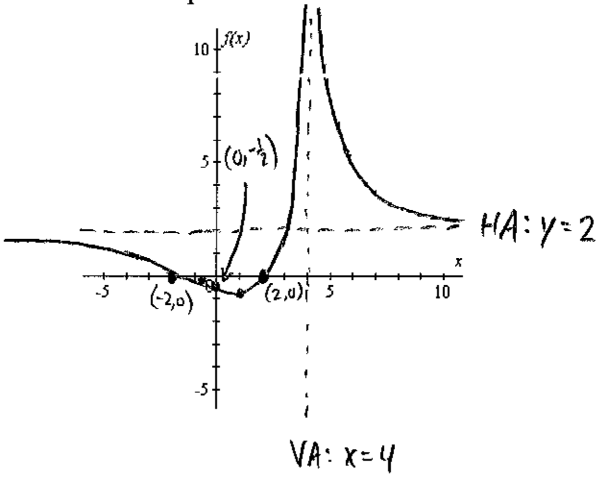
$f(0) = -\frac{8}{16} = -\frac{1}{2}$

VA: $(x-4)^2 = 0$

$x = 4$

HA: $\lim_{x \to -\infty} f(x) = 2$

$\lim_{x \to \infty} f(x) = 2$ → $y = 2$



3. Let $f(x) = x^{2/3} - 1$

a. Does the Mean Value Theorem apply on the interval $[0, 8]$? Justify your answer. If so, find the value(s) of c that satisfies the Mean Value Theorem over the interval.

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$ → f is not differentiable at $x = 0$

Since f is continuous on $[0,8]$ and differentiable on $(0,8)$, MVT applies.

$$\frac{\Delta f}{\Delta x} = \frac{f(8) - f(0)}{8 - 0} = \frac{3 - (-1)}{8} = \frac{1}{2}$$

$$\frac{2}{3x^{1/3}} = \frac{1}{2}$$

$$3x^{1/3} = 4$$

$$x^{1/3} = \frac{4}{3} \longrightarrow \boxed{x = \frac{64}{27}}$$

b. Does the Mean Value Theorem apply on the interval $[-1, 8]$? Justify your answer. If so, find the value(s) of c that satisfies the Mean Value Theorem over the interval.

Since f is not differentiable at $x = 0$ AND $x = 0$ is in the interval $(-1, 8)$, MVT does not apply.

4. Use the Second Derivative Test to find the value(s) of x corresponding to all relative extreme values for $f(x) = x^3 + 3x^2 - 9x$. Justify your answer(s).

$f' = 3x^2 + 6x - 9$

$3(x^2 + 2x - 3) = 0$

$(x+3)(x-1) = 0$

$x = -3, 1$

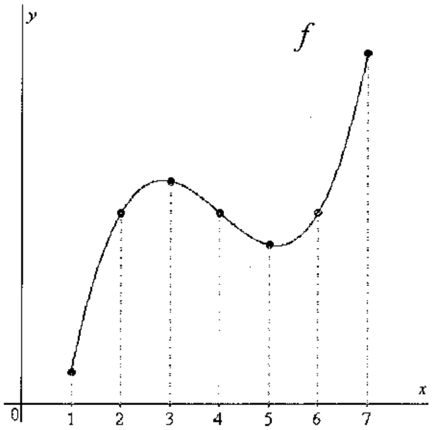
$f'' = 6x + 6$

$f''(-3) = -12 < 0 \rightarrow$ Relative Max @ $x = -3$

$f''(1) = 12 > 0 \rightarrow$ Relative Min @ $x = 1$

5. Use the graph at the right to complete the table.

Condition	Domain Interval/Value
$f'(x) < 0$	$(3,5)$
$f'(x) = 0$	$x = 3, x = 5$
* $f'(x) > 0$	$[1,3) \cup (5,7]$
$f''(x) < 0$	$(1,4)$
$f''(x) = 0$	$x = 4$
$f''(x) > 0$	$(4,7)$



* It would be OK if you gave $(1,3) \cup (5,7)$. When in doubt, leave the intervals open!!