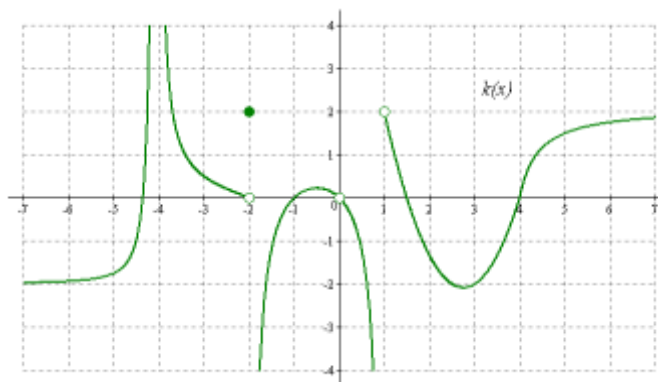


**AP CALCULUS AB**  
**Supplement 1.5**  
**Limits**

Name \_\_\_\_\_  
 Date \_\_\_\_\_  
 Period \_\_\_\_\_

1. Use the graph of  $k$  to answer the following.



a.  $\lim_{x \rightarrow -4^-} k(x) =$

b.  $\lim_{x \rightarrow -4^+} k(x) =$

c.  $\lim_{x \rightarrow -4} k(x) =$

d.  $\lim_{x \rightarrow -2^-} k(x) =$

e.  $\lim_{x \rightarrow -2^+} k(x) =$

f.  $\lim_{x \rightarrow -2} k(x) =$

g.  $\lim_{x \rightarrow 0^-} k(x) =$

h.  $\lim_{x \rightarrow 0^+} k(x) =$

i.  $\lim_{x \rightarrow 0} k(x) =$

j.  $\lim_{x \rightarrow 1^-} k(x) =$

k.  $\lim_{x \rightarrow 1^+} k(x) =$

l.  $\lim_{x \rightarrow 1} k(x) =$

m.  $\lim_{x \rightarrow 3^-} k(x) =$

n.  $\lim_{x \rightarrow 3^+} k(x) =$

o.  $\lim_{x \rightarrow 3} k(x) =$

p.  $\lim_{x \rightarrow 4^-} k(x) =$

q.  $\lim_{x \rightarrow 4^+} k(x) =$

r.  $\lim_{x \rightarrow 4} k(x) =$

s. Identify each horizontal asymptote for  $k$ . Describe each using one or more limits.

t. Identify each vertical asymptote for  $k$ . Describe each using one or more limits.

2. What does the following table suggest about the one-sided and two-sided limits as  $x$  approaches  $-1$  for the functions  $h(x)$ ,  $p(x)$ , and  $r(x)$ ?

$x$	$-1.1$	$-1.003$	$-1.0001$	$-0.9999$	$-0.8762$	$-0.6522$
$h(x)$	$89$	$677$	$5009$	$2.003$	$2.088$	$2.113$
$p(x)$	$16.222$	$16.111$	$16.002$	$15.999$	$15.802$	$15.777$
$r(x)$	$-99$	$-999$	$-9999$	$-8853$	$-871$	$-86$

3. Evaluate the following limits assuming that  $\lim_{x \rightarrow 2} f(x) = 6$  and  $\lim_{x \rightarrow 2} g(x) = 0$ .

a.  $\lim_{x \rightarrow 2} f(x)g(x)$

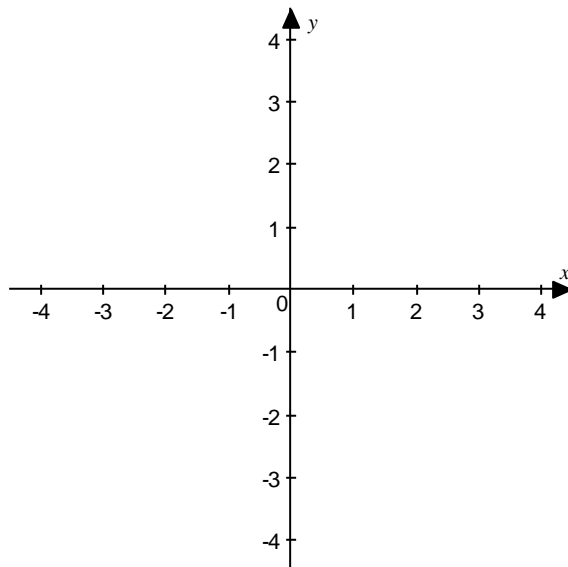
b.  $\lim_{x \rightarrow 2} \frac{f(x)}{g(x)}$

c.  $\lim_{x \rightarrow 2} x \cdot f(x)$

d.  $\lim_{x \rightarrow 2} \frac{\sqrt{g(x) + x^2}}{f(x)}$

4. Let  $g(x) = \begin{cases} -x & \text{if } x \leq -1 \\ 1-x^2 & \text{if } -1 < x < 1 \\ x-1 & \text{if } x > 1 \end{cases}$ .

a. Sketch a graph of  $g$ .



b. Evaluate the following limits.

i.  $\lim_{x \rightarrow -1^-} g(x)$

ii.  $\lim_{x \rightarrow -1^+} g(x)$

iii.  $\lim_{x \rightarrow -1} g(x)$

iv.  $\lim_{x \rightarrow 1^-} g(x)$

v.  $\lim_{x \rightarrow 1^+} g(x)$

vi.  $\lim_{x \rightarrow 1} g(x)$

Evaluate the following limits.

$$5. \lim_{z \rightarrow 1^+} \frac{z^2 - 1}{|z - 1|}$$

$$6. \lim_{v \rightarrow 5^-} \left( \frac{v^2 - 3v - 10}{|5 - v|} \right)$$

$$7. \lim_{y \rightarrow 2} \sin \left( \frac{y}{y - 2} \right)$$

$$8. \lim_{u \rightarrow 2} \cos^{-1} \left( \frac{\sqrt{2}}{u} \right)$$

$$9. \lim_{w \rightarrow 0} \frac{\frac{1}{6+w} - \frac{1}{6}}{w}$$

$$10. \lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x - 7}$$

$$11. \lim_{x \rightarrow 3} \frac{|3 - x|}{x - 3}$$

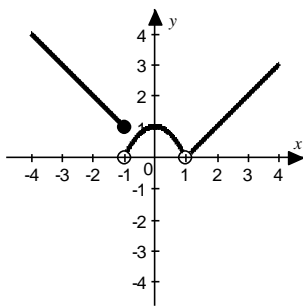
$$12. \lim_{v \rightarrow 0^+} \frac{6 - v^2}{v^3 - v}$$

### Supplement 1.5 Solutions

- 1a.  $\infty$                       1b.  $\infty$                       1c.  $\infty$   
 1d. 0                              1e.  $-\infty$                       1f. DNE  
 1g. 0                              1h. 0                              1i. 0  
 1j.  $-\infty$                       1k. 2                              1l. DNE  
 1m. -2                              1n. -2                              1o. -2  
 1p. 0                              1q. 0                              1r. 0
- 1s.  $y = -2, \lim_{x \rightarrow -\infty} k(x) = -2$                       1t.  $x = -4, \lim_{x \rightarrow -4} k(x) = \infty$   
 $y = 2, \lim_{x \rightarrow \infty} k(x) = 2$                                        $x = -2, \lim_{x \rightarrow -2^+} k(x) = -\infty$   
 $x = 1, \lim_{x \rightarrow 1^-} k(x) = -\infty$

2.  $\lim_{x \rightarrow -1^-} h(x) = \infty, \lim_{x \rightarrow -1^+} h(x) = 2, \lim_{x \rightarrow -1} h(x)$  DNE  
 $\lim_{x \rightarrow -1^-} p(x) = 16, \lim_{x \rightarrow -1^+} p(x) = 16, \lim_{x \rightarrow -1} p(x) = 16$   
 $\lim_{x \rightarrow -1^-} r(x) = -\infty, \lim_{x \rightarrow -1^+} r(x) = -\infty, \lim_{x \rightarrow -1} r(x) = -\infty$

3. a. 0                      b. DNE                      c. 12                      d. 1/3



4. a.  
 b. i. 1      ii. 0      iii. DNE      iv. 0      v. 0      vi. 0

5. 2                              6. -7                              7. DNE (rapid osc.)  
 8.  $\pi/4$                               9.  $-1/36$                               10. 1/6  
 11. DNE (one-sided limits disagree)                              12.  $-\infty$