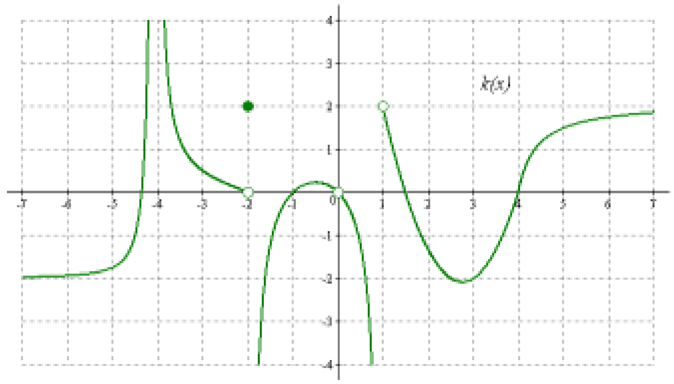
AP CALCULUS AB
Supplement 1.7
Continuity

Name ____________________
Date ____________________
Period ____________________

1. Use the graph of $k(x)$ to answer the following.

   a. Identify each discontinuity for k. Use the definition of continuity to justify each discontinuity. Classify each discontinuity as either *removable*, *jump*, or *infinite*.



   b. Redefine k so that it is continuous at each removable discontinuity.

2. Let $g(x) = \begin{cases} -x & \text{if } x \le -1 \\ 1 - x^2 & \text{if } -1 < x < 1 \\ x - 1 & \text{if } x > 1 \end{cases}$.

   a. Determine whether g is continuous at x = –2, –1, 0, 1, 2. Use the definition of continuity to justify each point of continuity and discontinuity. Classify each discontinuity as either *removable*, *jump*, or *infinite*. (Hint: You had to graph g on Supp 1.5.)

   b. Redefine g so that it is continuous at each removable discontinuity.

3. Suppose $f(x) = x^3 + 5$.

a. Show that $f(c) = 13$ for some c on the interval $(-1, 3)$. Find the value(s) of c.

b. Use IVT to verify that there is a root for $f(x)$ on the interval $(-2, 1)$.

4. Suppose the function h is continuous for all real numbers. Use the table below to answer the following questions.

x	0	1	2	3	4	5
$h(x)$	–12	–8	–7	1	3	–2

a. Suppose $g(x) = 3h(x) - 6$. Show that there must be a value n for $3 < n < 4$ such that $g(n) = 0$.

b. What is the fewest number of x-values for which h must equal 0? Explain your reasoning.

Supplement 1.7 Solutions

1a. $x = -4$; Justification: $\lim_{x \to -4} k(x) = \infty$ and $k(-4)$ is undefined → $\lim_{x \to -4} k(x) \neq k(-4)$; Classification: infinite

$x = -2$; Justification: $\lim_{x \to -2} k(x)$ DNE and $k(-2) = 2$ → $\lim_{x \to -2} k(x) \neq k(-2)$; Classification: infinite

$x = 0$; Justification: $\lim_{x \to 0} k(x) = 0$ and $k(0)$ is undefined → $\lim_{x \to 0} k(x) \neq k(0)$; Classification: removable

$x = 1$; Justification: $\lim_{x \to 1} k(x)$ DNE and $k(1)$ is undefined → $\lim_{x \to 1} k(x) \neq k(1)$; Classification: infinite

1b. $k(0) = 0$

2a. Continuous at $x = -2$; Justification: $\lim_{x \to -2} g(x) = 2$ and $g(-2) = 2$ → $\lim_{x \to -2} g(x) = g(-2)$

Discontinuous at $x = -1$; Justification: $\lim_{x \to -1} g(x)$ DNE and $g(-1) = 1$ → $\lim_{x \to -1} g(x) \neq g(-1)$; Classification: jump

Continuous at $x = 0$; Justification: $\lim_{x \to 0} g(x) = 1$ and $g(0) = 1$ → $\lim_{x \to 0} g(x) = g(0)$

Discontinuous at $x = 1$; Justification: $\lim_{x \to 1} g(x) = 0$ and $g(1)$ is undefined → $\lim_{x \to 1} g(x) \neq g(1)$; Classification: removable

Continuous at $x = 2$; Justification: $\lim_{x \to 2} g(x) = 1$ and $g(2) = 1$ → $\lim_{x \to 2} g(x) = g(2)$

2b. $g(1) = 0$

3a. Since *f* is a cubic function, *f* is continuous on $[-1, 3]$. Since $f(-1) = 4$ and $f(3) = 32$, we have $f(-1) < 13 < f(3)$. By IVT, there exists some *c* on the interval $(-1, 3)$ such that $f(c) = 13$.

3b. Since *f* is a cubic function, *f* is continuous on $[-2, 1]$. Since $f(-2) = -3$ and $f(1) = 6$, we have $f(-2) < 0 < f(1)$. By IVT, there exists some *c* on the interval $(-2, 1)$ such that $f(c) = 0$.

4a. Since *h* is continuous for all reals, *g* (which is a linear combination of *h*) is continuous on $[3, 4]$. Since $g(3) = 3h(3) - 6 = -3$ and $g(4) = 3h(4) - 6 = 3$, we have $g(3) < 0 < g(4)$. By IVT, there exists some *n* on the interval $(3, 4)$ such that $g(n) = 0$.

4b. The function h must equal zero for at least two values of x.

Since h is continuous for all reals, h is continuous on $[2, 3]$. Since $h(2) = -7$ and $h(3) = 1$, we have $h(2) < 0 < h(3)$. By IVT, there exists some c_1 on the interval $(2, 3)$ such that $h(c_1) = 0$. Since h is continuous for all reals, h is continuous on $[4, 5]$. Since $h(4) = 3$ and $h(5) = -2$, we have $h(5) < 0 < h(4)$. By IVT, there exists some c_2 on the interval $(4, 5)$ such that $h(c_2) = 0$.