

AP CALCULUS AB
Supplement 4.1
Rectilinear Motion

Name _____
Date _____
Period _____

1. (by hand) A particle moves along a straight line according to the position function $s(t) = \frac{1}{3}t^3 - t^2 - 15t$ for $t \geq 0$, where s is measured in feet and t is measured in seconds.
 - a. Find the velocity of the particle as a function of t and evaluate it at $t = 3$ seconds.
 - b. Find the acceleration of the particle at time t and evaluate it at $t = 4$ seconds.
 - c. When is the particle at rest? When is the particle moving forward? When is the particle moving backward?
 - d. When is the particle speeding up? When is the particle slowing down?
 - e. Find the total distance traveled by the particle on the intervals: (i) $[0, 3]$ and (ii) $[3, 6]$.
 - f. Calculate the position of the object each time the velocity is -15 ft/s.

2. (part (a) by hand, parts b – d with calculator) A particle moves along the x -axis, with position function $x(t) = \frac{t}{1+t^2}$ for $t \geq 0$, where x is measured in meters and t is measured in seconds.
 - a. Find the velocity and acceleration functions with respect to t .
 - b. When is the particle moving to the right? When is the particle moving to the left?
 - c. When is the particle speeding up? When is the particle slowing down?
 - d. Calculate the velocity of the object each time the acceleration is equal to -1 m/s².

3. A particle moves along a line according to the position function given in the graph below, where s is measured in cm and t is measured in seconds. Assume $\text{dom}(s) = [0, 6]$.

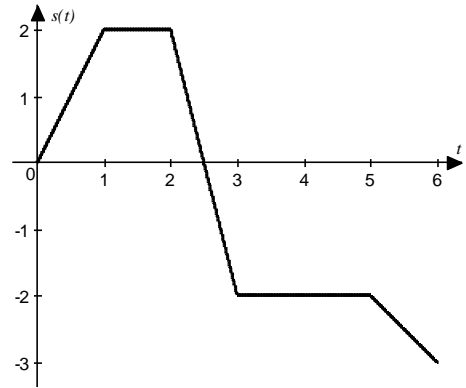
a. Find $s(5)$, $v(1.6)$, and $v(5.3)$.

b. When is the particle at rest?

c. When is the particle moving right? When is the particle moving left?

d. Find the value(s) of v each time the position of the particle is 1 cm.

e. Find the average velocity over the closed interval $[1, 6]$.



4. A particle moves along a line according to the velocity function given in the graph below, where v is measured in ft/min and t is measured in minutes. Assume $\text{dom}(v) = [0, 6]$.

a. Find $v(2)$, $a(2)$, and $a(4)$.

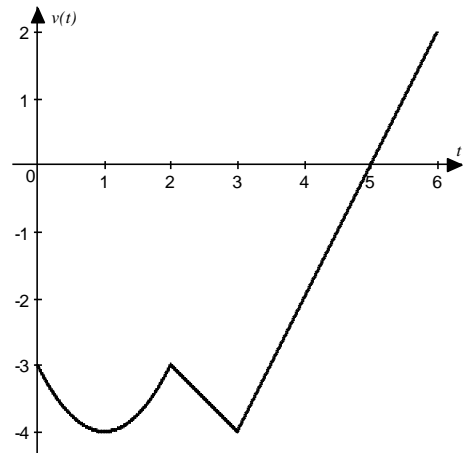
b. When is the particle at rest?

c. When is the particle moving right? When is the particle moving left?

d. When is the particle speeding up? When is the particle slowing down?

e. When does the particle move at its greatest speed?

f. Find the value(s) of a each time the velocity of the particle is -4 ft/min.



Supplement 4.1 Answers

- 1a. $v(t) = t^2 - 2t - 15$; $v(3) = -12$ ft/s
- 1b. $a(t) = 2t - 2$; $a(4) = 6$ ft/s²
- 1c. At rest: $t = 5$ s; Forward: $(5, \infty)$; Backward: $(0, 5)$
- 1d. Speeding Up: $(0, 1) \cup (5, \infty)$; Slowing Down: $(1, 5)$
- 1e. (i) 45 ft; (ii) $\frac{53}{3}$ ft
- 1f. $s(0) = 0$ ft; $s(2) = -\frac{94}{3}$ ft
- 2a. $v(t) = \frac{1-t^2}{(1+t^2)^2}$; $a(t) = \frac{2t(t^2-3)}{(1+t^2)^3}$
- 2b. Right: $(0, 1)$; Left: $(1, \infty)$
- 2c. Speeding Up: $(1, \sqrt{3})$; Slowing Down: $(0, 1) \cup (\sqrt{3}, \infty)$
- 2d. $v(0.187) = 0.901$ m/s; $v(0.730) = 0.199$ m/s
- 3a. $s(5) = -2$ cm; $v(1.6) = 0$ cm/s; $v(5.3) = -1$ cm/s
- 3b. $(1, 2) \cup (3, 5)$
- 3c. Right: $(0, 1)$; Left: $(2, 3) \cup (5, 6)$
- 3d. Position is 1 cm twice: $v(t_1) = 2$ cm/s; $v(t_2) = -4$ cm/s
- 3e. $v_{\text{ave}} = -1$ cm/s
 $a(4) = 2$ ft/min²
- 4a. $v(2) = -3$ ft/min; $a(2)$ undefined;
- 4b. $t = 5$ min
- 4c. Right: $(5, 6)$; Left: $(0, 5)$
- 4d. Speeding Up: $(0, 1) \cup (2, 3) \cup (5, 6)$; Slowing Down: $(1, 2) \cup (3, 5)$
- 4e. $t = 1$ min AND $t = 3$ min
- 4f. $a(1) = 0$ ft/min²; $a(3)$ undefined