

AP Calculus AB
Supplement 4.4
Linearization and Differentials

Name _____
Date _____
Period _____

1. Suppose that $f(-1) = 5$, $f'(-1) = 3$, and $f''(-1) = -4$. Using *linearization*, approximate $f(-1.5)$.
2. Suppose that $g(4) = 9$, $g'(4) = -1$, and $g''(4) = 1$. Using *linearization*, approximate $g(4.1)$.
3. Find the linearization of the given function at the specified value of a .
 - a. $y = \sqrt{2-x}$; $a = -7$
 - b. $h(x) = \cos 2x$; $a = \pi/6$
4. Use *linearization* to approximate each function value. Determine whether the approximations are overestimates or underestimates. Explain your reasoning.
 - a. $e^{0.2}$
 - b. $\ln 2$
5. Suppose a person measures the radius of a circle to be 10 in with a maximum error in measurement of 0.5 in. Use *differentials* to estimate the maximum error in the calculated area of the circle.
6. Suppose the diameter of a spherical balloon is measured as 10 cm. Use *differentials* to estimate the change in volume of the balloon if added helium increases the diameter to 12 cm.
7. Suppose a cylindrical metal rod of length 4 m and radius 2 m is subjected to a heating process that increases the length of the rod by 0.001 m. Assuming that the ratio of length to radius remains constant during the process, use *differentials* to estimate the increase in the surface area of the rod.

Supplement 4.4 Answers

1. $f(-1.5) \approx 3.5$

2. $g(4.1) \approx 8.9$

3a. $y(x) \approx 3 - \frac{1}{6}(x+7)$

3b. $h(x) \approx \frac{1}{2} - \sqrt{3}\left(x - \frac{\pi}{6}\right)$

4a. Using a linear approximation for $f(x) = e^x$ at $x = 0$:
 $e^{0.2} \approx 1.2$; underestimate since $f''(0) > 0$ on $[0, 0.2]$

4b. Using a linear approximation for $f(x) = \ln x$ at $x = 1$:
 $\ln(2) \approx 1$; overestimate since $f''(1) < 0$ on $[1, 2]$

5. Maximum Error $\approx dA = 10\pi \text{ in}^2$

6. $\Delta V \approx dV = 100\pi \text{ cm}^3$

7. $\Delta S \approx dS = 0.037699 \text{ m}^2$