AP Calculus AB
Supplement 5.6
Net Change and Rectilinear Motion

Name ____________________
Date ____________________
Period ____________________

1. A bicycle manufacturer asks an intern to determine the current bicycle inventory. The intern knows that 1087 bicycles were in inventory at the beginning of the most recent 12-hour work shift, and was provided with the following table of values, which gives production rates $R(t)$, measured in bicycles per hour, at select times *t* during this 12-hour shift.

t	0	2	5	6	10	11	12
$R(t)$	8	6	10	11	7	6	4

a. The intern determines that $\int_0^{12} R(t)\,dt$ is an important computation. In the context of the problem, describe the meaning of $\int_0^{12} R(t)\,dt$ using appropriate units.

b. The intern decides to use a left Riemann sum with the six subintervals indicated by the data to approximate the number of bicycles currently in inventory. Find this approximation.

c. To obtain a better approximation, the intern decides to use a trapezoidal sum with the six subintervals indicated by the data to approximate the number of bicycles currently in inventory. Find this approximation.

d. Finally, the intern uses the data to find a quadratic regression for *R*:

$$R(t) = 6.581 + 1.241t - 0.119t^2$$

Approximate the number of bicycles currently in inventory using this model.

2. Suppose a particle is moving along a linear path, and its velocities (in m/s) at different times are recorded in the table below. Assuming its position function $x(t)$ is twice-differentiable with a given position of $x(2) = 3$ m, answer the following.

t	0	2	3	6	8	9	10
$v(t)$	2	6	7	5	2	–1	–3

a. Use a right Riemann sum with the six subintervals indicated by the data to approximate the value of $\int_0^{10} v(t)\,dt$.

b. Using appropriate units, describe the meaning of the value found in (a).

c. Use a trapezoidal sum with the five subintervals indicated by the data to approximate the position of the particle at 10 sec.

d. Use (i) a right Riemann sum and (ii) a trapezoidal sum with the six subintervals indicated by the data to approximate the distance traveled by the particle on the interval $0 \le t \le 10$.

After careful analysis, the function $v(t) = 7.209\sin(0.325t + 0.317)$ is suggested as an appropriate model for the velocity of the particle.

e. Using the model above, find the position of the particle at 10 sec.

f. Using the model above, find the distance traveled by the particle on the time interval $0 \le t \le 10$.

Supplement 5.6 Answers

1a. The $\int_0^{12} R(t)\,dt$ represents the number of bicycles produced during the 12-hour shift.

1b. $1087 + \int_0^{12} R(t)\,dt \approx 1188$ bicycles

1c. $1087 + \int_0^{12} R(t)\,dt \approx 1183$ bicycles

1d. $1087 + \int_0^{12} R(t)\,dt = 1186.78$ → If prompted to round to the nearest whole number, the final answer would be approximately 1187 bicycles.

2a. $\int_0^{10} v(t)\,dt \approx 34\,\text{m}$

2b. The particle has been *displaced* approximately 34 meters over the time interval $t = 0$ to $t = 10$ seconds.

2c. $x(10) = 3 + \int_2^{10} v(t)\,dt \approx 33\,\text{m}$

2d. (i) $d = \int_0^{10} |v(t)|\,dt \approx 42\,\text{m}$; (ii) $d = \int_0^{10} |v(t)|\,dt \approx 43\,\text{m}$

2e. $x(10) = 3 + \int_2^{10} v(t)\,dt = 35.798562\,\text{m}$

2f. $d = \int_0^{10} |v(t)|\,dt = 45.234903\,\text{m}$