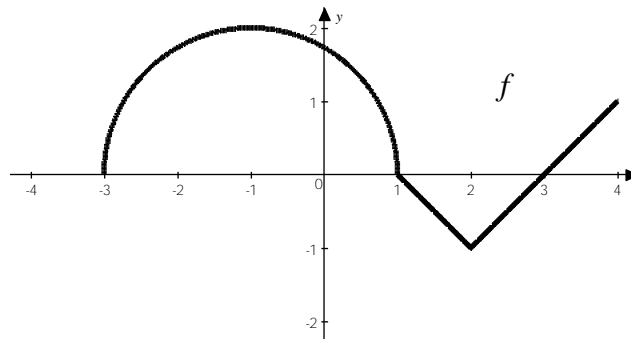


**AP Calculus AB**  
**Supplement 5.9**  
**Integration Review**

Name \_\_\_\_\_  
Date \_\_\_\_\_  
Period \_\_\_\_\_

1. Consider the functions  $g(x) = \int_1^x f(t) dt$  and  $h(x) = \int_{-1}^{x^2} f(t) dt$ . Use the graph of  $f$  to answer the following.



a. Find  $g(-3)$ .

b. Find  $g(1)$ .

c. Find  $g(4)$ .

d. Find  $g'(2)$ .

e. Find  $g''(3)$ .

f. Find  $h(2)$ .

g. Find  $h'(2)$ .

h. Find the intervals on  $(-3, 4)$  such that  $g$  is increasing and decreasing.

i. Find the intervals on  $(-3, 4)$  such that  $g$  is concave up and concave down.

2. The acceleration of a particle traveling along a horizontal path is given by  $a(t) = 2t - 8$  for  $t \geq 0$ , where  $a$  is in  $\text{m/s}^2$ . The velocity of the particle at  $t = 0$  is  $-20 \text{ m/s}$ , and the position of the particle at  $t = 1$  second is  $32 \text{ m}$ .

a. Find the velocity as a function of time and the velocity at  $t = 4$ .

b. Find the displacement of the particle over the time interval  $[0, 15]$ .

c. Find the total distance traveled by the particle over the time interval  $[0, 15]$ .

d. Determine the final position of the particle at  $t = 15$ .

e. What is the position of the particle when the acceleration is zero?

f. What is the position of the particle when the particle is at rest?

3. Find the derivative of the function  $y = \int_{\pi}^{\sin x} \sqrt{t} dt$ .

4. Evaluate the following indefinite integrals.

a.  $\int \frac{9x \, dx}{\sqrt{1-x^2}}$

b.  $\int \cot(3x) \, dx$

c.  $\int \frac{\sec^2(1/x^3)}{x^4} \, dx$

d.  $\int \frac{dx}{\sqrt{9-x^2}}$

e.  $\int \frac{e^\theta}{\sqrt{1-e^{2\theta}}} \, d\theta$

f.  $\int \frac{x-1}{x^2+3} \, dx$

5. Evaluate the following definite integrals.

a.  $\int_0^{\pi/3} \cos x(2 + \sin^2 x) dx$

b.  $\int_2^3 \frac{x}{1+x^2} dx$

c.  $\int_0^1 x\sqrt{1-x} dx$

d.  $\int_1^2 (x-4^x) dx$

e.  $\int_0^3 |1-x^2| dx$

f.  $\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x} dx$

### Supplement 5.9 Answers

1a.  $-2\pi$

1b. 0

1c.  $-\frac{1}{2}$

1d. -1

1e. 1

1f.  $\pi - \frac{1}{2}$

1g. 4

1h. Increasing:  $(-3, 1) \cup (3, 4)$ ; Decreasing:  $(1, 3)$

1i. Concave Up:  $(-3, -1) \cup (2, 4)$ ; Concave Down:  $(-1, 1) \cup (1, 2)$

2a.  $v(t) = t^2 - 8t - 20$ ;  $v(4) = -36$  m/s

2b. -75 m

2c.  $\frac{1375}{3}$  m

2d.  $x(15) = -\frac{58}{3}$  m

2e.  $x(4) = -67$  m

2f.  $x(10) = -211$  m

3.  $y' = \cos(x)\sqrt{\sin(x)}$

4a.  $-9\sqrt{1-x^2} + C$

4b.  $\frac{1}{3} \ln|\sin(3x)| + C$

4c.  $-\frac{1}{3} \tan\left(\frac{1}{x^3}\right) + C$

4d.  $\arcsin\left(\frac{x}{3}\right) + C$

4e.  $\sin^{-1}(e^\theta) + C$

4f.  $\frac{1}{2} \ln(x^2 + 3) - \frac{1}{\sqrt{3}} \arctan\left(\frac{x}{\sqrt{3}}\right)$

5a.  $\frac{9\sqrt{3}}{8}$

5b.  $\frac{\ln 2}{2}$

5c.  $\frac{4}{15}$

5d.  $\frac{3}{2} - \frac{12}{\ln(4)}$

5e.  $\frac{22}{3}$

5f.  $\frac{\pi}{4}$