1.1 – The Limit of a Function

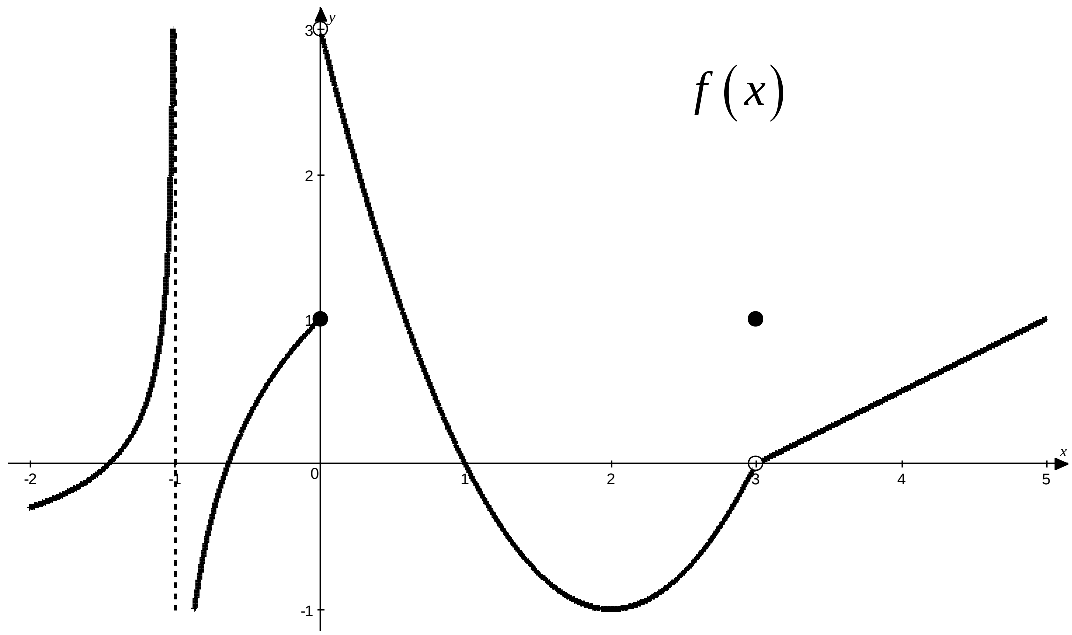
Introduction to Limits

The limit is the most fundamental operation of the calculus. To find a limit, we determine the *behavior* of *f* near $x = a$, whether or not $f(a)$ is defined. The notation for limits is as follows:

- Two-sided: $\lim_{x \to a} f(x) = L$
- One-sided from the left: $\lim_{x \to a^-} f(x) = L$
- One-sided from the right: $\lim_{x \to a^+} f(x) = L$

In essence, we are saying that the values of *f* get closer and closer to *L* as *x* approaches *a*.

1. Use the graph of f below to find each limit.



a. $\lim\limits_{x \to 1} f(x)$

b. $\lim\limits_{x \to 3^-} f(x)$

c. $\lim\limits_{x \to 3^+} f(x)$

d. $\lim\limits_{x \to 3} f(x)$

e. $\lim\limits_{x \to 0^-} f(x)$

f. $\lim\limits_{x \to 0^+} f(x)$

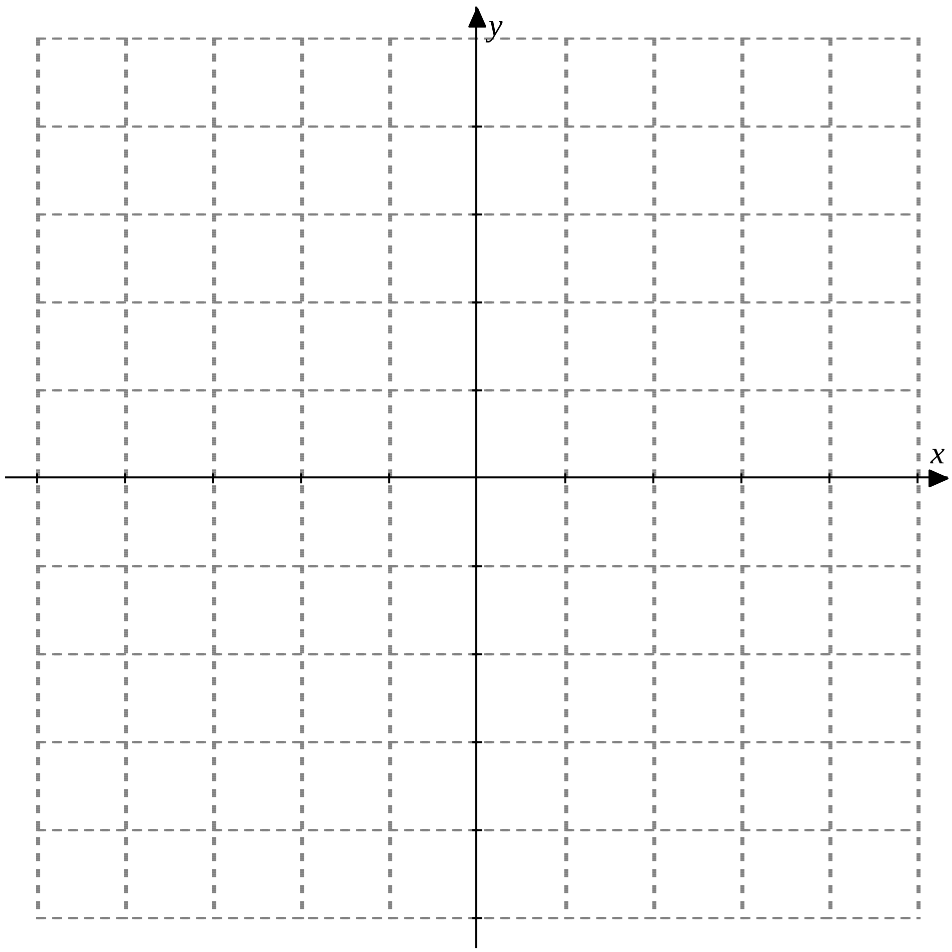
g. $\lim\limits_{x \to 0} f(x)$

h. $\lim\limits_{x \to -1^-} f(x)$

i. $\lim\limits_{x \to -1^+} f(x)$

j. $\lim\limits_{x \to -1} f(x)$

2. Sketch a graph of the function $g(x)=\begin{cases} x, & \text{if } x<-2 \\ 2-x^2, & \text{if } -2<x\le 2. \\ 5-2x, & \text{if } x>2 \end{cases}$



Use the graph above to determine the following limits:

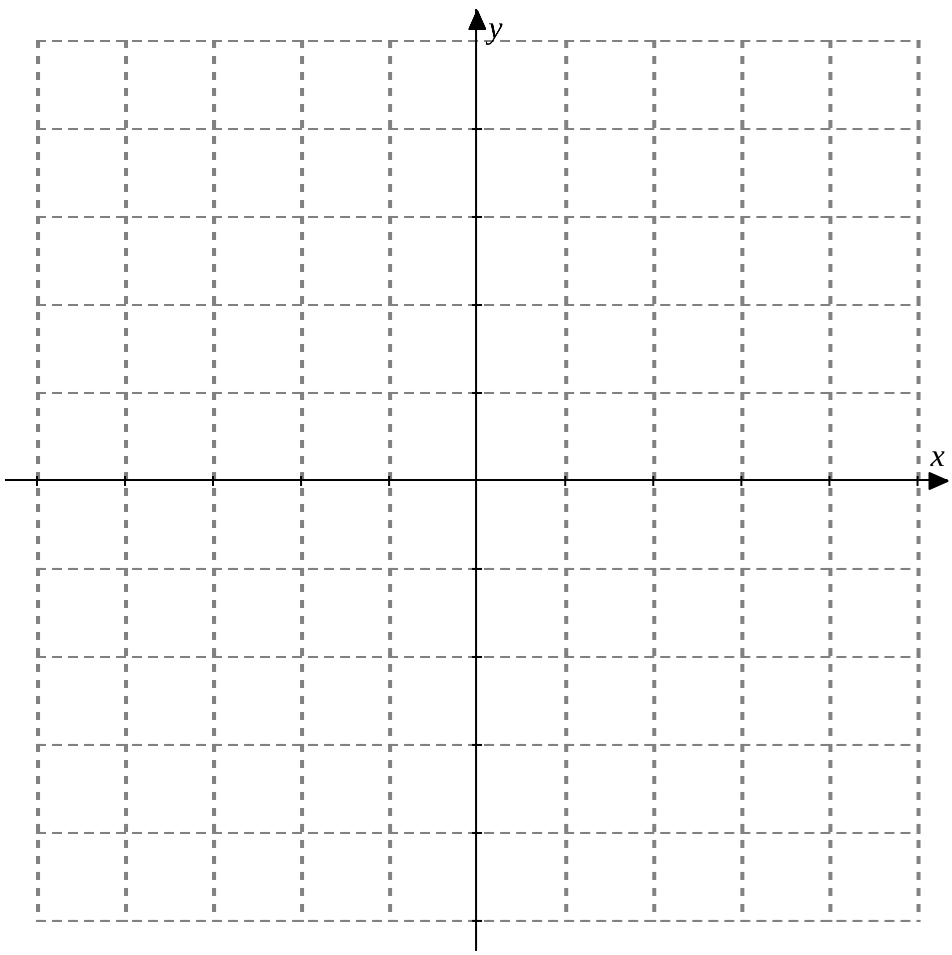
a. $\lim_{x\to -2} g(x)$

b. $\lim_{x\to 2} g(x)$

3. Sketch the graph of a function *f* that satisfies the following:

$$\lim_{x \to 0^-} f(x) = 1, \ \lim_{x \to 0^+} f(x) = -2, \ f(0) = -1,$$

$$\lim_{x \to 3} f(x) = -\infty, \ f(3) = 5, \ f(-1) \text{ is undefined}$$



4. Estimate each limit by completing the corresponding table of values.

a. $\lim_{x \to -1} \frac{\sin(x+1)}{x+1} \approx$

x	–1.1	–1.01	–0.99	–0.9
$\frac{\sin(x+1)}{x+1}$				

b. $\lim_{x \to 3^-} \frac{x^2}{3-x} \approx$

x	2.9	2.99	3.01	3.1
$\frac{x^2}{3-x}$				

c. $\lim_{x \to 0} \cos\left(\frac{4}{x}\right) \approx$

x	–0.1	–0.01	0.01	0.1
$\cos\left(\frac{4}{x}\right)$				