1.2 – Finding Limits Analytically, Part I

Assume that $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist. Then,

- **Sum Law:** The limit of a sum is the sum of the limits.

$$\lim_{x \to a}\left(f(x) + g(x) \right) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

- **Constant Multiple Law:** Constant multiples may be factored out of a limit.

$$\lim_{x \to a} k\, f(x) = k \lim_{x \to a} f(x), \text{ where } k \in \mathbb{R}$$

- **Product Law:** The limit of a product is the product of the limits.

$$\lim_{x \to a}\left(f(x) g(x) \right) = \lim_{x \to a} f(x) \lim_{x \to a} g(x)$$

- **Quotient Law:** The limit of a quotient is the quotient of the limits. If $\lim_{x \to a} g(x) \neq 0$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$$

1. Given that $\lim_{x\to 3} f(x) = -4$, $\lim_{x\to 3} g(x) = 5$, and $\lim_{x\to 3} h(x) = 6$, calculate the following limits:

   a. $\lim_{x\to 3} 5f(x)g(x)$

   b. $\lim_{x\to 3} \frac{g(x)}{3h(x)}$

   c. $\lim_{x\to 3} \left(h(x) - 5x + 6\right)$

   d. $\lim_{x\to 3} \frac{x^{-2} + f(x)g(x)}{3\left(x - 5h(x)\right)}$

2. Find the following limits.

a. $\lim_{\theta \to \pi} e^{\cos\theta}$

b. $\lim_{x \to 4} \frac{x^2 + 2x - 24}{x^3 - 4x^2}$

c. $\lim_{x \to 0} \frac{\sqrt{x^2 + 16} - 4}{x^2}$

d. $\lim_{z \to 5} \dfrac{z - \sqrt{z+20}}{z-5}$

e. $\lim_{h \to 0} \dfrac{\dfrac{1}{(a+h)^2} - \dfrac{1}{a^2}}{h}$, where a is a constant

f. Challenge Problem: $\lim_{x \to 4} \dfrac{\sqrt{5-x} - 1}{2 - \sqrt{x}}$