1.3 – Finding Limits Analytically, Part II

1. Consider the function $g\left(x\right)=\dfrac{x^2-16}{\left|x-4\right|}$.

   a. Re-write g as a piecewise-defined function.

   b. Find $\lim_{x\to 4} g\left(x\right)$. Use two-sided limits to justify your answer.

2. Use the procedure from #1 to find the following limits.

a. $\lim\limits_{x \to 5^-} \dfrac{x^4 - 5x^3}{|5 - x|}$

b. $\lim\limits_{u \to 9^+} \dfrac{x - 9}{|x^2 - 7x - 18|}$

Composition Rule for Limits

If $\lim_{x \to a} g(x) = L$ and $f(L)$ is defined, then

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a}(g(x))\right) = f(L)$$

3. $\lim_{x \to 3} \ln\left(\frac{x-3}{x^2-9}\right)$

4. $\lim_{x \to 1} \arcsin\left(\frac{1-\sqrt{x}}{1-x}\right)$

5. $\lim_{x \to 0} e^{\frac{1}{x^2}}$

6. $\lim_{x \to 0^+} |\ln x|$

The Squeeze Theorem

If $f(x) \le g(x) \le h(x)$ when x is near a and

$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$, then $\lim_{x \to a} g(x) = L$.

7. If $3 - 2x + x^2 \le r(x) \le 1 + 2x - x^2$ for all x, find $\lim_{x \to 1} r(x)$.

8. Use the Squeeze Theorem to find $\lim_{x \to 0} x^2 \cos\left(\frac{4}{x}\right)$.