1.4 – Limits Involving Infinity, Part I

Definition of Vertical Asymptote

The line $x = a$ is a vertical asymptote of f if

$$\lim_{x \to a^-} f(x) = \infty \text{ or } -\infty$$

AND/OR

$$\lim_{x \to a^+} f(x) = \infty \text{ or } -\infty$$

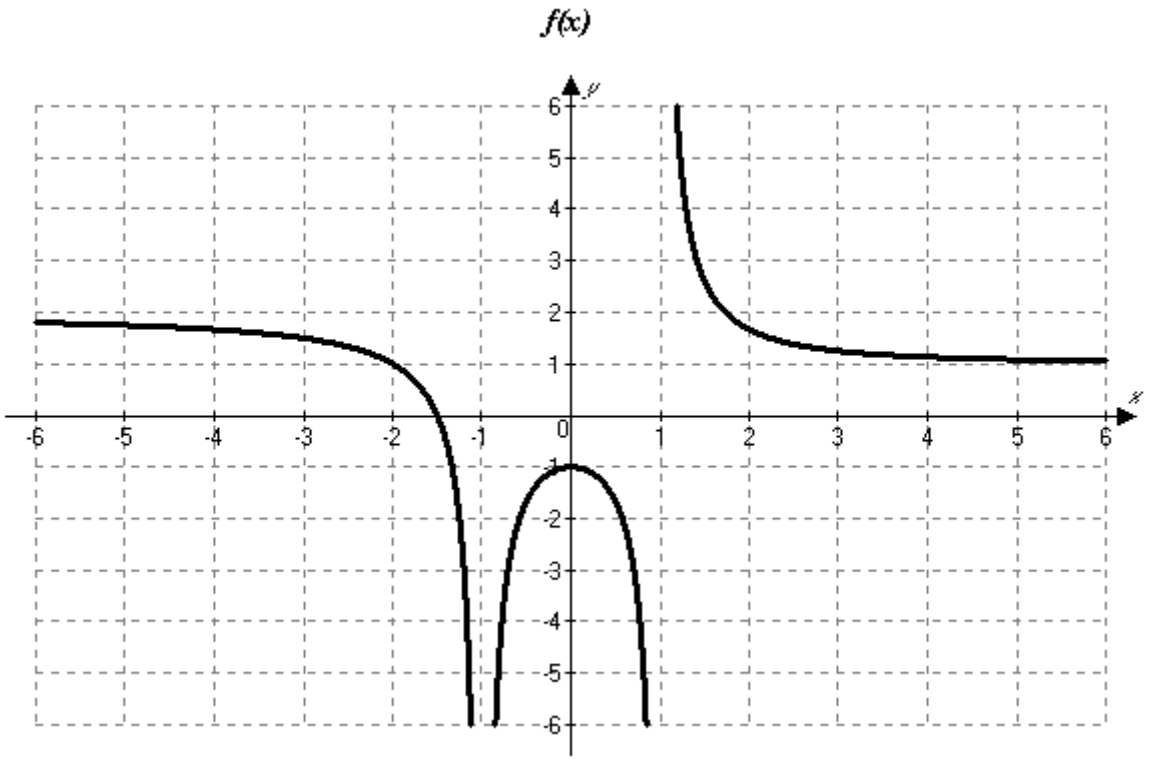
Definition of Horizontal Asymptote

The line $y = b$ is a horizontal asymptote of f if

$$\lim_{x \to \infty} f(x) = b$$

AND/OR

$$\lim_{x \to -\infty} f(x) = b$$

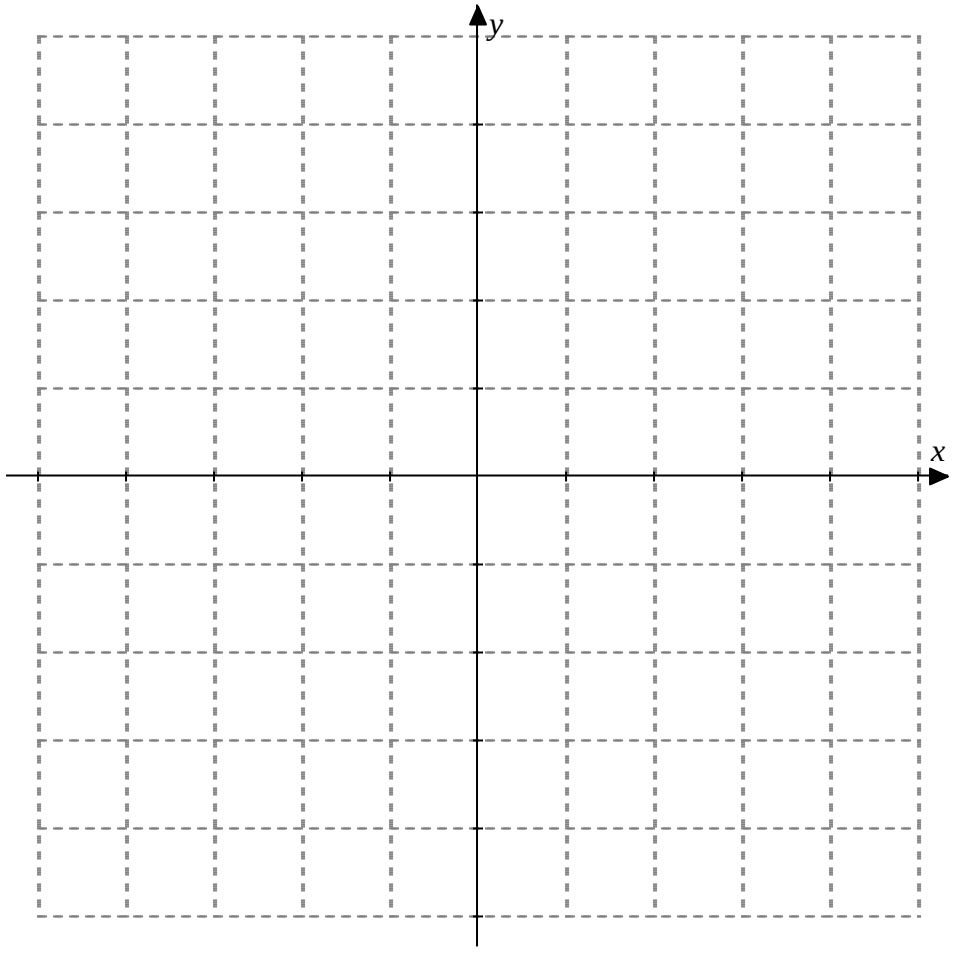


1. Use the graph given above to identify all horizontal and vertical asymptotes of *f*. Describe each using limits.

2. Sketch the graph of a function *f* that satisfies the following:

$$\lim_{x\to-\infty} f(x) = -3,\ \lim_{x\to-1} f(x) = -\infty,\ \lim_{x\to 2^-} f(x) = \infty,$$

$$f(2) = 1,\ \lim_{x\to 2^+} f(x) = -1,\ \lim_{x\to\infty} f(x) = -\infty$$



Infinite Limits

3. Let $g(x) = \dfrac{4 - x^2}{3 - x}$.

   a. $\lim\limits_{x \to 3^-} g(x)$

   b. $\lim\limits_{x \to 3^+} g(x)$

   c. Based on the results from (a) and (b), what can we determine about the graph of *g*?

4. Find the following limits. What does the value of the limit indicate about the graph of each function?

a. $\lim_{x \to 3^-} \dfrac{x-5}{x^2-9}$

b. $\lim_{\theta \to \pi/2^+} \tan\theta$