1.6 – Continuity

<u>Limit Definition of Continuity</u>

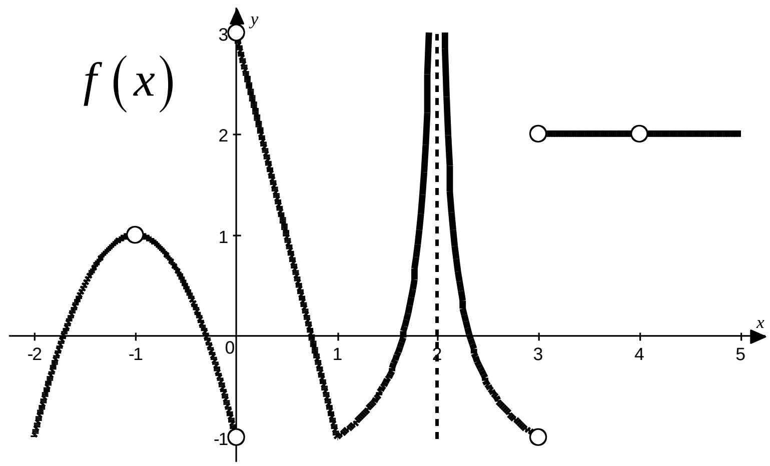
A function *f* is continuous at $x = a$ if

- $f(a)$ is defined
- $\lim_{x \to a} f(x)$ exists
- $\lim_{x \to a} f(x) = f(a)$

** Roughly speaking, a function is said to be continuous if it is connected. Can you trace the graph of the function without lifting your finger?

** A function *f* is **<u>right-continuous</u>** at $x = a$ if the above is true for limits from the right and **<u>left-continuous</u>** if the above is true for limits from the left.

1. Use the graph of *f* to answer the following.



a. For what values of *x* is the function $f(x)$ discontinuous? Justify your answers.

b. Classify each discontinuity above as either a *removable*, *jump*, or *infinite* discontinuity.

c. Redefine *f* so that it is continuous at each removable discontinuity.

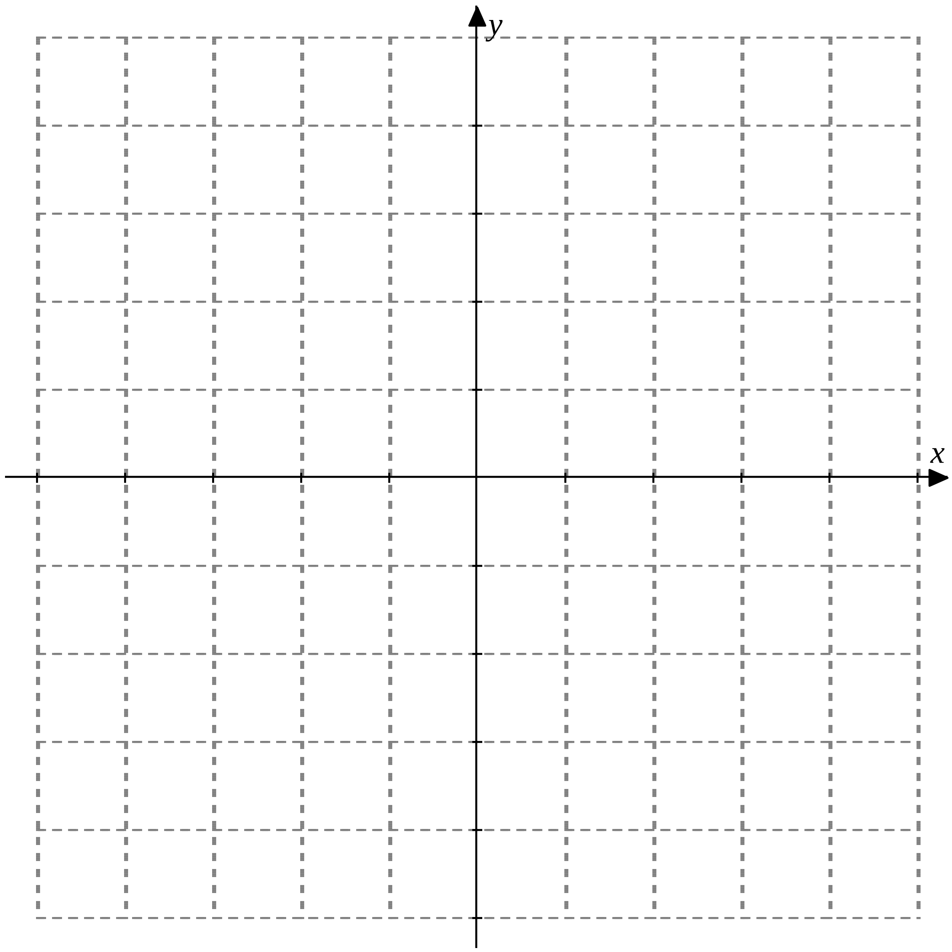
2. For what value(s) of x are the following functions discontinuous? Justify each answer and classify each discontinuity.

a. $f\left(x\right)=\dfrac{x-3}{x^2-9}$

b. $y=\dfrac{\left|x+2\right|}{x+2}$

c. $r\left(\theta\right)=\tan\theta$

3. Sketch a graph of $g(x)=\begin{cases}\cos(\pi x), & \text{if } x<-2 \\ -x-1, & \text{if } -2<x<1 \\ \ln x, & \text{if } x\geq 1\end{cases}$.



For what value(s) of x is g discontinuous? Justify each answer and classify each discontinuity.

4. Find the value of c such that $h(x) = \begin{cases} 8 - cx, \text{ if } x < -5 \\ 4x^2 + 3c, \text{ if } x \geq -5 \end{cases}$ is continuous for all real numbers.

5. Determine whether the following functions are continuous at $x = c$. Justify each answer.

a. $f(x) = \begin{cases} x^2 - 2x, & x \leq 3 \\ 10 - 3x, & x > 3 \end{cases}$

b. $g(x) = \begin{cases} \ln(x+2), & x < -1 \\ \sin(\pi x), & x \geq -1 \end{cases}$