2.5 – The Chain Rule, Part II

The derivative of a general exponential function is given by

$$\frac{d}{dx}\left[a^x\right] =$$

1. Differentiate.

   a. $y = e^{4x}\cos\left(x^4\right)$

   b. $f(v) = 6\tan\left(\dfrac{v}{v^2 - 4}\right)$

c. $r = \sin^2(9\theta)$

d. $h(x) = 3x^2 + 3^{x^2}$

2. Find an equation for the line tangent to the curve
$P = \dfrac{100}{1 + 24e^{-0.5t}}$ at $t = 0$.

3. Use the table of values below to find the derivative of each function at the given value of x.

x	0	1	2
$f(x)$	−4	5	−2
$g(x)$	−10	2	3
$f'(x)$	−6	−3	4
$g'(x)$	5	8	9

a. $y = \dfrac{x^2}{f(x)}; \ x = 2$

b. $y = g(f(x) - 3x); \ x = 1$

c. $y = f(x)\sqrt{g(x)}$; $x = 2$

Chain Rule as a Change of Variables

The Chain Rule can also be applied when differentiation problems require a change of variables, as given by

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$$

4. Suppose $y = \sin(2x)$, where x and y are function of t. Find $\left.\frac{dy}{dt}\right|_{t=2}$ given that $x(2) = 3$ and $\left.\frac{dx}{dt}\right|_{t=2} = -5$.

5. You work for the Empire at a manufacturing plant on Coruscant. The cost of producing hyperdrive motivators is given by $C(h) = 7h^3 + 6h^2 + 102$, where C is cost in Imperial Credits and h is number of hyperdrive motivators produced. After 3 hours, your crew has produced 118 hyperdrive motivators, and is currently producing at a rate of 50 hyperdrive motivators per hour. What is the cost of producing hyperdrive motivators per hour for your crew at this time?