2.8 – Derivatives of Inverse Functions

1. Find the derivative of $f^{-1}(x)$ at $x=8$ given $f(x)=2x^3$.

2. Unfortunately, not all functions have an inverse (or the inverse is very difficult to find). Develop a formula for the derivative of an inverse function that is independent of the inverse function itself.

3. Use the formula derived in #2 to find $\left(g^{-1}\right)'(2)$ given $g(x) = x - \sin x$.

4. Develop formulas for $\frac{d}{dx}\left[\arcsin x\right]$ and $\frac{d}{dx}\left[\tan^{-1} x\right]$.

5. Calculate the derivative for the following functions.

a. $f\left(x\right)=x^{3}\sin^{-1}2x$

b. $y=\arcsin\sqrt{x}$

c. $g\left(u\right)=\left(\arctan u\right)^{5}$

d. $x=\tan^{-1}\left(\cos y\right)$

6. Use the following table to find the derivative of each at the given value of x.

x	–1	0	1	2
$f(x)$	0	6	2	–1
$g(x)$	5	1	0	1
$f'(x)$	3	1	–3	–2
$g'(x)$	–6	–2	1	0

a. $y = f^{-1}(x);\ x = 2$

b. $y = g^{-1}(x);\ x = 5$

c. $y = g^{-1}(f(x));\ x = -1$