3.1 – The Mean Value Theorem and Introduction to the Shape of a Curve

<u>The Mean Value Theorem (MVT)</u>

If a function *f* is continuous on $[a, b]$ and differentiable on (a, b), then there exists some value *c* on (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Rolle's Theorem is a special case of the MVT such that the average rate of change over $[a, b]$ is zero.

1. Verify that each of the following meet the conditions of the MVT. Determine the value(s) of *c* guaranteed by the MVT (or Rolle's Theorem) for each given function and interval.

   a. $f(x) = 4\sqrt{2x - 1}$; $[1, 5]$

b. $g(x) = x^4 - 2x^2$; $[-2, 2]$

c. $y = \tan^{-1} x$; $[-1, 0]$

2. Does the MVT apply to the function $y = \dfrac{x}{3-x}$ on the interval $[-2, 4]$? Justify your answer.

3. Does the MVT apply to the function $y = 6 - |x+1|$ on the interval $[-5, 1]$? Justify your answer.

<u>Increasing/Decreasing Intervals</u>

Suppose *f* is continuous on [*a*, *b*] and differentiable on (*a*, *b*).

- If $f' > 0$ for all *x* on (*a*, *b*), then *f* is **<u>increasing</u>** on (*a*, *b*).

- If $f' < 0$ for all *x* on (*a*, *b*), then *f* is **<u>decreasing</u>** on (*a*, *b*).

<u>Definition of a Critical Number</u>

A **<u>critical number</u>** is an *x*-value on the domain of a function *f* such that f' is either zero or undefined.

<u>First Derivative Test</u>

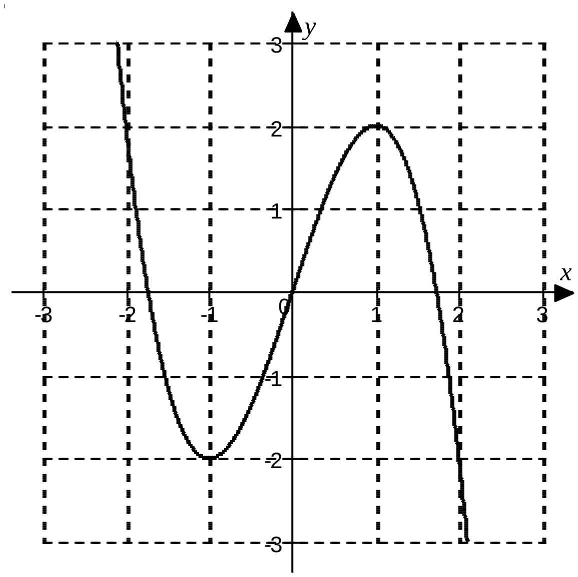
Suppose that *c* is a critical number for *f* and that *f* is differentiable for all values of *x* on some interval containing *c*, except possibly at $x = c$.

- If f' changes from positive to negative at *c*, then *f* has a **<u>relative (local) maximum</u>** at *c*.

- If f' changes from negative to positive at *c*, then *f* has a **<u>relative (local) minimum</u>** at *c*.

4. Consider the graph of *f* given below. Answer and justify the following.

a. On what intervals is *f* increasing/decreasing?

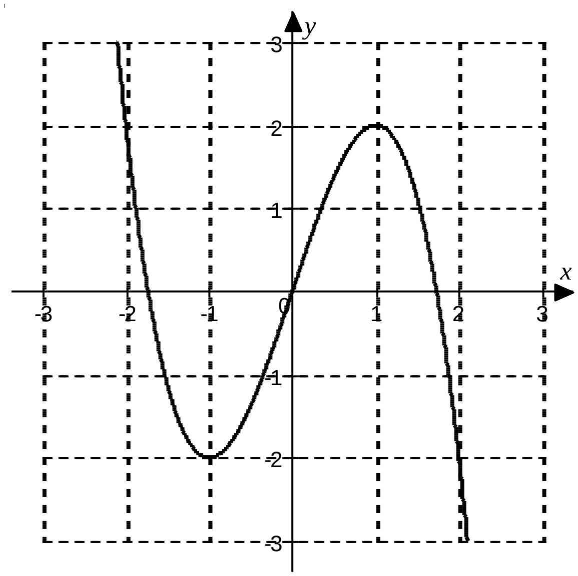


b. At what value(s) of *x* does *f* have a relative maximum?

c. At what value(s) of *x* does *f* have a relative minimum?

5. Consider the graph of f' given below. Answer and justify the following.

   a. On what intervals is *f* increasing/decreasing?



   b. At what value(s) of *x* does *f* have a relative maximum?

   c. At what value(s) of *x* does *f* have a relative minimum?