3.3 – What does f'' Say About f?

Intervals of Concavity

Suppose f is a twice-differentiable function.

- If $f'' > 0$ for all x on (a, b), then f is **concave up** on (a, b).
- If $f'' < 0$ for all x on (a, b), then f is **concave down** on (a, b).

Points of Inflection

The point $(a,\ f(a))$ is a **point of inflection** if:

- $f(a)$ and $f'(a)$ are defined;
- $f''(a)$ is either zero or undefined; **AND**
- concavity changes for the graph of f at $x = a$.

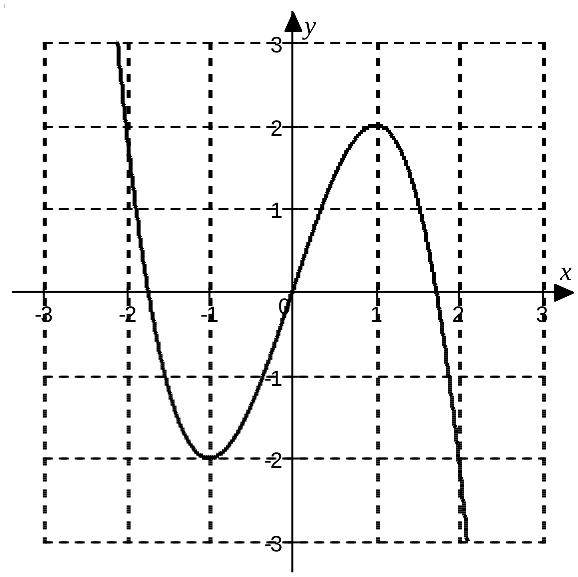
Second Derivative Test

Suppose f'' is continuous on some open interval containing $x = c$.

- If $f'(c) = 0$ and $f''(c) < 0$, then f has a **relative maximum** at $x = c$.
- If $f'(c) = 0$ and $f''(c) > 0$, then f has a **relative minimum** at $x = c$.
- If $f'(c) = 0$ and $f''(c) = 0$, then the test is inconclusive.

1. Consider the graph of *f* given below. Answer and justify the following.

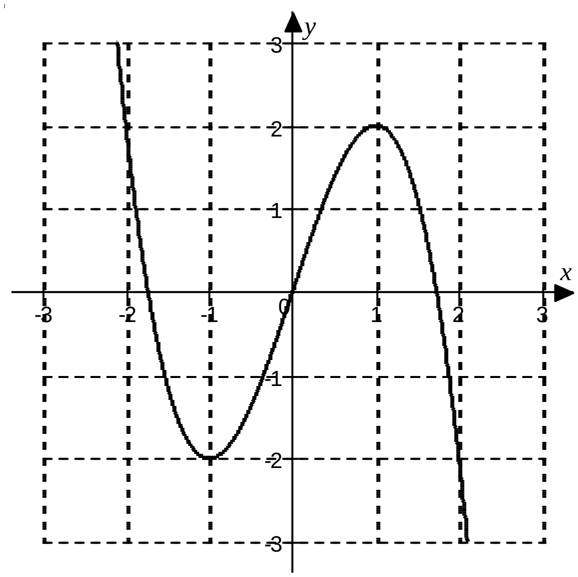
   a. What are the intervals of concavity for *f*?



   b. At what value(s) of x does *f* have a point of inflection?

2. Consider the graph of f' given below. Answer and justify the following.

a. What are the intervals of concavity for f?



b. At what value(s) of x does f have a point of inflection?

3. Find the intervals of concavity and points of inflection for the following.

   a. $h(x) = 5x^3 - 3x^5$

   b. $y = x^2 \ln x$

c. $h(x)=\dfrac{x}{(x+1)^2}$

4. Use the Second Derivative Test to determine all relative extreme values for the function $y=x^2e^x$.