3.4 – L’Hôpital’s Rule, Part I

We will now shift our focus to the analysis of points of discontinuity. L’Hôpital’s Rule provides an alternate method for determining limits of indeterminate forms.

Indeterminate Forms

Some common indeterminate forms that often result from direct substitution applied to a limit are:

$$\frac{0}{0},\ \frac{\infty}{\infty},\ 0\cdot\infty,\ \infty-\infty,\ 0^0,\ 1^\infty,\ \infty^0$$

L’Hôpital’s Rule

Suppose *f* and *g* are differentiable on an open interval containing *a*. If direct substitution into $\lim\limits_{x\to a}\frac{f(x)}{g(x)}$ yields either $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

Evaluate the following limits.

1. $\lim_{x\to 0} \frac{x+\tan x}{\sin 6x}$

2. $\lim_{x\to \infty} \frac{e^x}{x^3}$

3. $\lim_{x\to 0^+} \frac{\ln x}{x}$

4. $\lim_{x\to 0} \frac{\tan(x) - x}{x^3}$

5. $\lim_{x\to -\infty} \frac{e^x}{x^2}$

6. $\lim_{x\to \infty} \frac{\ln x}{\sqrt[3]{x}}$

7. $\lim_{x \to -\infty} x^2 e^{4x}$

8. $\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right)$

9. $\lim_{x \to 0^+} x^3 \ln x$