4.2 – Related Rates, Part I

Relates rates problems involve scenarios in which multiple aspects of a physical phenomenon vary instantaneously with respect to time. Since the variables are typically not defined explicitly with respect to time, we apply the change of variables form of *chain rule*:

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$$

to each variable, which means each variable is derived *implicitly* with respect to time.

1. Air is being pumped into a spherical balloon at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when diameter is 50 cm?

2. A 17-ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5 ft/s, how fast will the top of the ladder be moving down the wall when it is 8 ft above the ground?

3. A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving due east. When the cruiser is 1.6 miles north of the intersection and the car is 1.2 miles east of the intersection, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 65 mph at the instant of measurement, what is the speed of the car?

4. Water is flowing out of the bottom of an inverted conical tank at the rate of 100 ft^3/hr. The base diameter of the tank is 10 ft and the height of the tank is 20 ft. How fast is the water level falling when the water is 8 ft deep?

5. Sand falls from a chute at a rate of 40 cm^3/min onto the top of a conical pile. The height of the pile is always equal to the base diameter of the pile. How fast is the height of the pile changing when the height is 8 cm?