4.4 – Linear Approximations and Differentials

Linear Approximation is a method of approximating the value of a function using a tangent to the function near the point of approximation. The formula for linearization is given by

$$f(x) \approx$$

1. Use a linear approximation to estimate $f(1.3)$ given $f(1) = -2$, $f'(1) = 4$, and $f''(1) = -8$.

2. Let $g(x) = \arctan(4x)$. Find the linear approximation of g at $x = 1/4$ and use it to estimate $g(1/2)$. Is the estimate an overestimate or underestimate? Explain your reasoning.

3. Find the linear approximation of $h(x) = \sqrt[3]{8 - x}$ at $a = 0$ and use it to estimate $\sqrt[3]{7.96}$. Is the estimate an overestimate or underestimate? Explain your reasoning.

In contrast to linearization, *differentials* approximate the change in function value. The formula for differentials is given by

$$\Delta f \approx$$

4. A square plate with side length 4 cm undergoes mechanical processing that causes each side to increase 0.002 cm. Use differentials to estimate the change in the area of the square plate.

5. An intern uses a set of calipers to measure the diameter of a ball bearing as 2 mm. The head engineer determines that the intern may have made an error of as much as 0.2 mm in the measurement. Use differentials to estimate the maximum error in the volume of the ball bearing if we use the intern's measurement. What is the relative error? What is the percent error?