5.2 – The Definite Integral

The first few HW problems for this lesson provide you with further opportunities to practice Riemann sum approximations.

General Summation Rules

- Sum and Difference Rule:

$$\sum_{k=1}^{n}\left(a_k \pm b_k\right) = \sum_{k=1}^{n} a_k \pm \sum_{k=1}^{n} b_k$$

- Constant Multiple Rule:

$$\sum_{k=1}^{n} c\,a_k = c\sum_{k=1}^{n} a_k$$

- Constant Summation Rule:

$$\sum_{k=1}^{n} c = n \cdot c$$

- Sum of First *n* Integers:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

- Sum of First *n* Squares:

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

To find the definite integral, which computes the **exact** area under the curve defined by *f* over the interval [*a*, *b*], we calculate the limit of an infinite Riemann sum:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} \Delta x\, f(x_k)$$

where $\Delta x = \dfrac{b-a}{n}$ and $x_k = a + k\Delta x$. The above formula yields the *limiting value* of a right-endpoint Riemann sum.

1. Use the definition of the integral to compute $\int_1^3 \left(3x^2 - x + 5\right)dx$.

Creating a Bound for Definite Integrals

If $m \le f(x) \le M$ for $a \le x \le b$, then

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$

2. Find a bound for the value of $\int_0^1 e^{-x^2}\,dx$.

3. Find a bound for the value of $\int_1^e \ln x\,dx$.

Discuss the concept of signed, net area under the curve using Figures 3 and 4 on page 373.

4. Calculate the following definite integrals using geometric area formulae.

   a. $\int_{-2}^{2} \sqrt{4-x^2}\, dx$

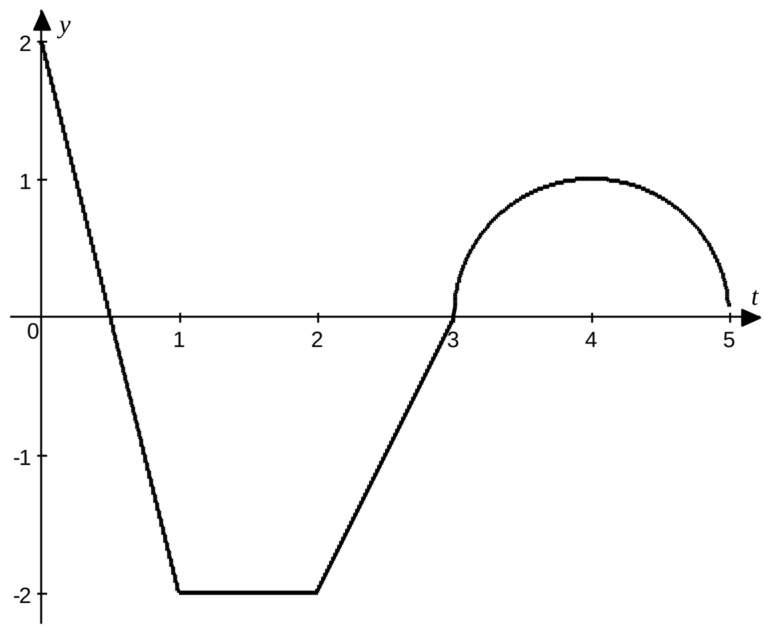
   b. $\int_{-2}^{6} (3-|x|)\, dx$

Properties of Definite Integrals

If *f* and *g* are continuous on the closed interval [*a*, *b*] and $c \in \mathbb{R}$, then

- $\int_a^b \left(f(x) + g(x) \right) dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
- $\int_a^b c\, f(x)\,dx = c\int_a^b f(x)\,dx$
- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
- $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

5. Use the graph of *f* given below to find the given integrals.



a. $\int_{0}^{3} f(t)\,dt$

b. $\int_{5}^{2} f(t)\,dt$