5.5 – The Fundamental Theorem of Calculus (FTC), Part 1

FTC 1

Suppose $g(x) = \int_a^{u(x)} f(t)\,dt$ is continuous on [a, b] and differentiable on (a, b). Then

$$g'(x) = \frac{d}{dx}\int_a^{u(x)} f(t)\,dt = f\left(u(x)\right)\cdot u'(x)$$

1. Find $\frac{dy}{dx}$ for the function $y = \int_1^{x^2} \left(1 - t^3\right)dt$ in two ways:
   (a) Solve for y using FTC 2 and derive, and (b) apply FTC 1.

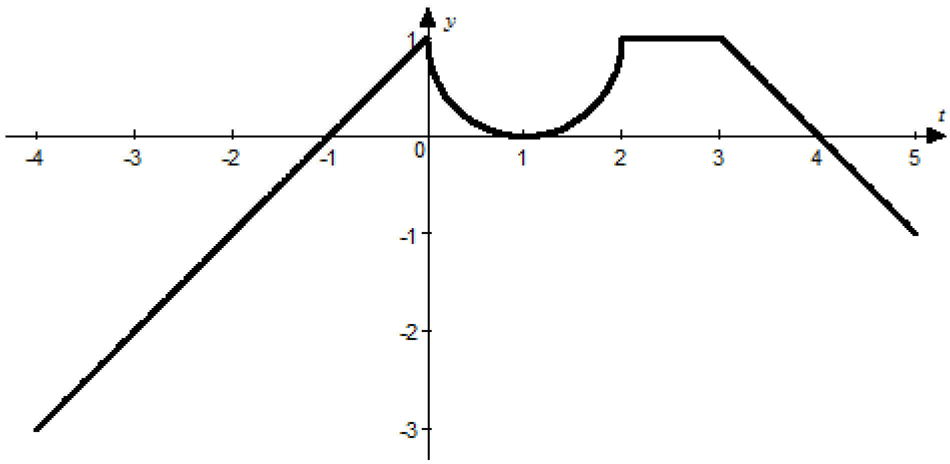
2. Find the derivative of the following functions.

a. $g(x) = \int_1^x \sqrt[3]{t-5}\,dt$

b. $f(x) = \int_{\tan x}^4 \ln t\,dt$

c. $r(y) = \int_{3y}^{4-y^2} \log u\,du$

3. Suppose $f(x)=\int_2^x y(t)\,dt$. Use the graph of y given below to answer the following.



a. Evaluate $f(-4)$, $f(2)$, and $f(5)$.

b. Evaluate $f'(-2)$, $f'(0)$, $f'(2.683)$, and $f'(4)$.

c. Evaluate $f''(1)$, $f''(2)$, $f''(2.683)$, and $f''(4)$.

d. Determine the intervals of increase/decrease for *f*. At what <u>point(s)</u> does *f* attain relative and absolute extreme values?

e. Determine the intervals of concavity for *f*. For what <u>value(s) of *x*</u> does have *f* have point(s) of inflection?