5.6 – Total Change

Total Change of a Quantity

Suppose that Q and R are continuous on $[a, b]$ and R is differentiable on (a, b). If R is a rate function and Q is the resulting quantity function (i.e., $R = Q'$), then the change in Q over the interval $[a, b]$ is defined by

$$\Delta Q = \int_a^b R(t)\,dt$$

Given an initial quantity $Q(a)$, the final quantity obtained over the interval $[a, b]$ is defined by

$$Q(b) = Q(a) + \Delta Q = Q(a) + \int_a^b R(t)\,dt$$

1. During a 10-hour manufacturing process, the temperature of a rod varies according the function $H(t) = 0.005e^{\frac{t}{2}} \sin\left(\frac{t}{2}\right)$, where H is measured in $°\text{C} / \text{hr}$ and t is measured in hours. The initial temperature of the metal rod was $37\ °C$.

   a. Using appropriate units, describe the meaning of $\int_0^{10} H(t)\,dt$ in the context of this problem.

   b. Find the temperature of the rod at $t = 8$ hours.

   c. At what time and temperature is the temperature of the rod (i) at a maximum, and (ii) at a minimum?

Rectilinear Motion

Suppose a, v, and s represent the acceleration, velocity, and position of an object moving along a linear path, respectively. Since $a = v'$ and $v = s'$, we have

- Velocity at time t: $v(t) = v(a) + \int_a^t a(u)\,du$

- Position at time t: $s(t) = s(a) + \int_a^t v(u)\,du$

Three important quantities for an object moving along a linear path over a closed interval $[a, b]$ are as follows:

- Displacement: $\int_a^b v(t)\,dt$

- Distance: $\int_a^b |v(t)|\,dt$

- Final Position: $s(b) = s(a) + \int_a^b v(t)\,dt$

2. An object moves along a horizontal line with position function $x(t)$. The velocity of the object is given by $v(t) = 3t^2 + 6t - 24$ for $t \geq 0$, where v is measured in ft/s and t is measured in seconds. Given that $x(1) = -10$, answer the following.

   a. Find the distance travelled by the object on the interval $[0, 3]$.

   b. Find the position of the object at $t = 4$ seconds.

2. (continued)

   c. Find the position of the object as a function of *t*.

   d. On the interval $[1, 5]$, find the time and position of the object when it is (i) farthest left, and (ii) farthest right.