

6.3 – Volumes by Cross-Sectional Area

Cross-Sectional Area Perpendicular to the x -axis

If the cross-sectional area of a solid bounded by $a \leq x \leq b$ and perpendicular to the x -axis can be defined by a continuous function $A(x)$, then the volume of the solid is

$$V = \int_a^b A(x) dx$$

Cross-Sectional Area Perpendicular to the y -axis

If the cross-sectional area of a solid bounded by $a \leq y \leq b$ and perpendicular to the y -axis can be defined by a continuous function $A(y)$, then the volume of the solid is

$$V = \int_a^b A(y) dy$$

Consider the region bounded by the curves $y = 2x^2$ and $y = 8$.
Find the volume of the solid if:

1. cross sections perpendicular to the x -axis are squares

2. cross sections perpendicular to the y -axis are semicircles

3. cross sections perpendicular to the x -axis are equilateral triangles

4. cross sections perpendicular to the y -axis are isosceles right triangles with one leg on the xy -plane