6.4 – Volumes of Solids of Revolution, Part I

<u>**Rotating About a Fixed Horizontal Line**</u>

Suppose a solid S is formed by rotating a region in the *xy*-plane about a fixed horizontal line.

- <u>Disk</u>: If the cross-sectional area of S bounded by $a \le x \le b$ is a *circle* of radius $R(x)$, then $A(x) = \pi\left(R(x)\right)^2$. So, the volume of S is

$$V = \int_a^b A(x)\,dx = \int_a^b \pi\left(R(x)\right)^2\,dx$$

- <u>Washer</u>: If the cross-sectional area of S bounded by $a \le x \le b$ is an *annulus* such that $R(x) < r(x)$ for all $x \in [a, b]$, then $A(x) = \pi\left(R(x)\right)^2 - \pi\left(r(x)\right)^2$. So, the volume of S is

$$V = \int_a^b A(x)\,dx = \int_a^b \pi\left(R(x)\right)^2\,dx - \int_a^b \pi\left(r(x)\right)^2\,dx$$

Rotating About a Fixed Vertical Line

Suppose a solid S is formed by rotating a region in the *xy*-plane about a fixed vertical line.

- Disk: If the cross-sectional area of S bounded by $a \le y \le b$ is a *circle* of radius $R(y)$, then $A(y) = \pi\left(R(y)\right)^2$. So, the volume of S is

$$V = \int_a^b A(y)\,dy = \int_a^b \pi\left(R(y)\right)^2 dy$$

- Washer: If the cross-sectional area of S bounded by $a \le y \le b$ is an *annulus* such that $R(y) < r(y)$ for all $y \in [a, b]$, then $A(y) = \pi\left(R(y)\right)^2 - \pi\left(r(y)\right)^2$. So, the volume of S is

$$V = \int_a^b A(y)\,dy = \int_a^b \pi\left(R(y)\right)^2 dy - \int_a^b \pi\left(r(y)\right)^2 dy$$

1. Consider the region R bounded by the curves $y = e^{-2x}$, $y = 0$, $x = 0$, and $x = 2$.

   a. Find the volume of the solid formed when R is revolved about the x-axis.

   b. Find the volume of the solid formed when R is revolved about the y-axis.

2. Consider the region D bounded by the curves $x = y^2 - 6$, $x = 0$, and $y = 0$ such that $y \leq 0$.

   a. Find the volume of the solid formed when D is revolved about the y-axis.

   b. Find the volume of the solid formed when D is revolved about the x-axis.

3. Consider the region Φ bounded by the curves $x = y^2$ and $y = x^2$. Find the volume of the solid formed when Φ is revolved about the y-axis.

4. Consider the region Ω bounded by the curves $y = e^{-2x}$ and $y = 2 - x^2$. Find the volume of the solid formed when Ω is revolved about the x-axis.