6.7 – Solving Differential Equations by Separation of Variables

Separable differential equations are of the form

$$\frac{dy}{dx} = f(x)g(y)$$

These equations are called "separable" because it is possible to separate the variables and re-write the equation as

$$\frac{1}{g(y)}dy = f(x)dx$$

We can now use integration to solve for *y*, which is the primary goal of all differential equations.

1. Find the general solution for each differential equation.

   a. $\frac{dy}{dx} = \frac{2x}{y^4}$

b. $y \ln x - x \dfrac{dy}{dx} = 0$

c. $P' = 2 + 2P + t + tP$

2. Find the particular solution for each differential equation with given initial condition.

a. $\frac{dy}{d\theta} = \frac{y^2 + 1}{\sec\theta};\ y\left(\frac{\pi}{2}\right) = 0$

b. $x^3 y' = y^2;\ y(2) = 4$

3. Find the particular solution for the differential equation $xy = 2x - y'$ given the initial condition $y(2) = 1$. What is the limiting value of $y(x)$?