

Using derivatives to graph a function without a calculator #2

$$f(x) = \frac{x-5}{(x-1)^2}$$

a. Find the x intercepts. (Give answer as a coordinate)

(5, 0)

b. Find the y intercepts (Give answer as a coordinate)

(0, -5)

c. Find any asymptotes (horizontal and vertical). State the type of asymptote and the equation.

V.A.  $x=1$

H.A.  $y=0$

V.A.  $\lim_{x \rightarrow 1} f(x) = -\infty$

d. Find the end behavior. State answer as a limits.

(H.A.)  $\lim_{x \rightarrow \infty} f(x) = 0$

$\lim_{x \rightarrow -\infty} f(x) = 0$

e. Find the first derivative (get a common denominator).

$$f'(x) = \frac{(x-1)^2 - (x-5)(2)(x-1)}{(x-1)^4} = \frac{x-1-2x+10}{(x-1)^3}$$

$$= \frac{(x-1)[x-1-2(x-5)]}{(x-1)^4} = \frac{9-x}{(x-1)^3}$$

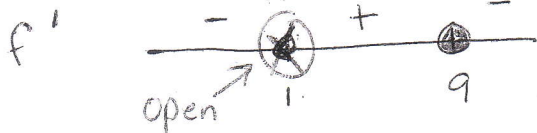
f. Find the critical points. (hint: set numerator=0 and denominator = 0)

$9-x=0$   
 $x=9$

$(x-1)^3=0$

$x=1$ , but is a V.A., so not a critical number

g. Use the critical points to find any max/mins. (hint: use a sign line)



local max at  $x=9$   
 $f(9) = \frac{1}{16}$

$(9, \frac{1}{16})$  local max

h. State intervals of increase and decrease.

$f$  is increasing  $(1, 9)$

$f$  is decreasing  $(-\infty, 1) \cup (9, \infty)$

i. Find the second derivative (get a common denominator).

$$f''(x) = \frac{(x-1)^3(-1) - (9-x)3(x-1)^2}{(x-1)^6} = \frac{-x+1-27+3x}{(x-1)^4}$$

$$= \frac{2x-26}{(x-1)^4} = \frac{2(x-13)}{(x-1)^4}$$

j. Find all possible points of inflection. (hint: numer = 0 and denom = 0)

$$x-13=0$$

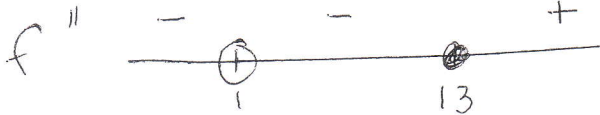
$$(x-1)^4=0$$

$$x=13$$

$x=1$ , but is a V.A.

$$\left(13, \frac{8}{144}\right)$$

k. Find intervals of concavity. (hint: use a sign line)



$f$  is concave up  $(13, \infty)$

$f$  is concave down  $(-\infty, 1) \cup (1, 13)$

l. Sketch the graph of  $f(x)$ . Label intercepts, asymptotes, and max/mins.

