AP Calculus AB **Name**________________________

Using derivatives to graph a function without a calculator #3

$$f(x) = \frac{x^2 - 1}{x^2 - 9}$$

a. Find the x intercepts. $x^2 - 1 = 0$ $x = \pm 1$ $(1, 0)$ $(-1, 0)$

b. Find the y intercepts $f(0) = \frac{1}{9}$ $(0, \frac{1}{9})$

c. Find any asymptotes (horizontal and vertical)

V.A $x = \pm 3$ H.A $y = 1$

d. Find the end behavior. $\lim_{x \to \infty} f(x) = 1$ $\lim_{x \to -\infty} f(x) = 1$

V.A. $\lim_{x \to 3^+} f(x) = \infty$

$\lim_{x \to 3^-} f(x) = -\infty$

$\lim_{x \to -3^+} f(x) = -\infty$

$\lim_{x \to -3^-} f(x) = +\infty$

e. Find the first derivative (get a common denominator).

$$f' = \frac{(x^2-9)2x - (x^2-1)2x}{(x^2-9)^2}$$

$$= \frac{2x(x^2-9-x^2+1)}{(x^2-9)^2} = \frac{2x(-8)}{(x^2-9)^2} = \boxed{\frac{-16x}{(x^2-9)^2}}$$

f. Find the critical points. (hint: set numerator=0 and denominator = 0)

$x = 0$ $x = \pm 3$ V.A

g. Use the critical points to find any max/mins. (hint: use a sign line)

f' + (-3) + 0 − (3) −

local max $(0, \frac{1}{9})$

h. State intervals of increase and decrease.

f increasing $(-\infty, -3) \cup (-3, 0)$

f decreasing $(0, 3) \cup (3, \infty)$

$$f' = \frac{-16x}{(x^2-9)^2}$$

i. Find the second derivative (get a common denominator).

$$f'' = \frac{(x^2-9)^2(-16) - (-16x)\,2(x^2-9)\,2x}{(x^2-9)^4}$$

$$= \frac{\cancel{(x^2-9)}(-16)[x^2-9-4x^2]}{(x^2-9)^{\cancel{4}\,3}} = \frac{-16(x^2-4x^2-9)}{(x^2-9)^3} = \frac{-16(-3x^2-9)}{(x^2-9)^3} = \boxed{\frac{48(x^2+3)}{(x^2-9)^3}}$$

j. Find all possible points of inflection. (hint: numer = 0 and denom = 0)

$x^2+3=0$ No soln

$x^2-9=0$ $x=\pm 3$ (VA)

No ppoi , No poi

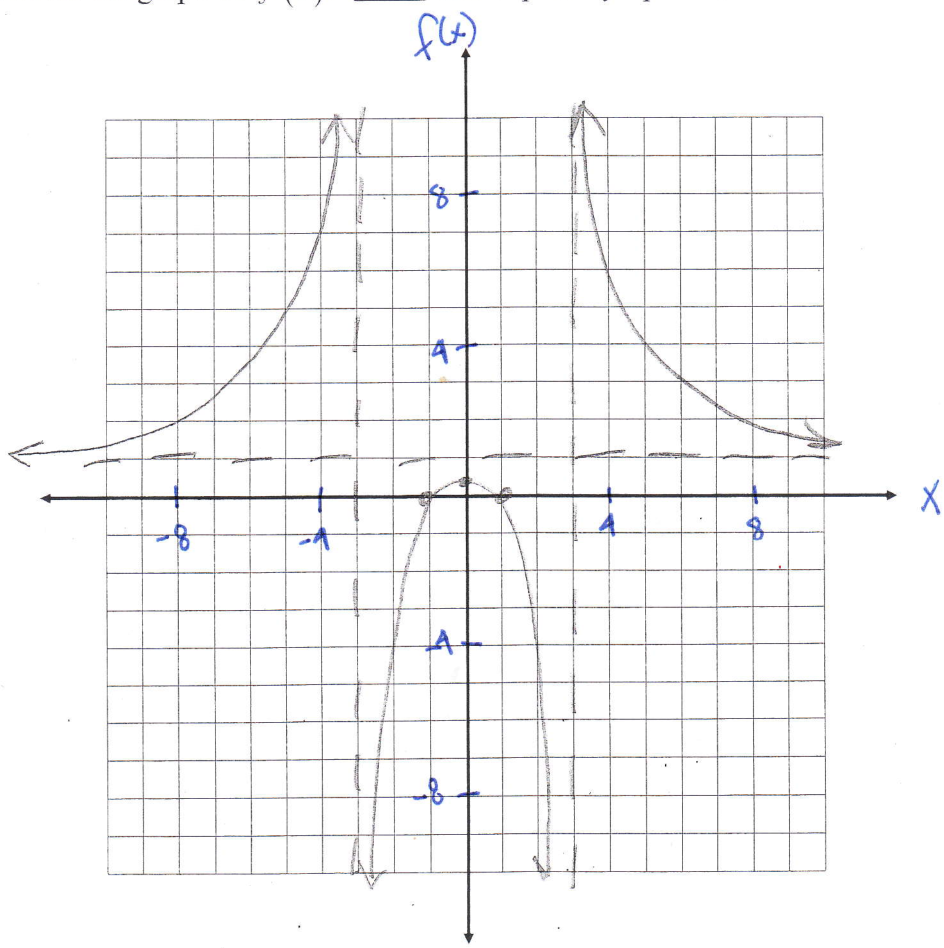
k. Find intervals of concavity. (hint: use a sign line)

f'': + (−3) − (3) +

Concave up $(-\infty,-3)\cup(3,\infty)$

Concave down $(-3,3)$

l. Sketch the graph of $f(x)$. Label intercepts, asymptotes, and max/mins.



AP Calculus AB **Name**________________________________

Using derivatives to graph a function without a calculator #3

$$f(x)=\frac{x^2-1}{x^2-9}$$

a. Find the x intercepts. $x^2-1=0$ $x=\pm 1$ $(1,0)$ $(-1,0)$

b. Find the y intercepts $f(0)=\frac{1}{9}$ $\left(0,\frac{1}{9}\right)$

c. Find any asymptotes (horizontal and vertical)

V.A $x=\pm 3$ H.A $y=1$

V.A. $\lim_{x\to 3^+} f(x)=\infty$

$\lim_{x\to 3^-} f(x)=-\infty$

$\lim_{x\to -3^+} f(x)=-\infty$

$\lim_{x\to -3^-} f(x)=+\infty$

d. Find the end behavior. $\lim_{x\to\infty} f(x)=1$ $\lim_{x\to -\infty} f(x)=1$

e. Find the first derivative (get a common denominator).

$$f'=\frac{(x^2-9)2x-(x^2-1)2x}{(x^2-9)^2}=\frac{2x(x^2-9-x^2+1)}{(x^2-9)^2}=\frac{2x(-8)}{(x^2-9)^2}=\boxed{\frac{-16x}{(x^2-9)^2}}$$

f. Find the critical points. (hint: set numerator=0 and denominator = 0)

$x=0$ $x=\pm 3$ V A

g. Use the critical points to find any max/mins. (hint: use a sign line)

f': + (−3) + (0) − (3) −

local max $\left(0,\frac{1}{9}\right)$

h. State intervals of increase and decrease.

f increasing $(-\infty,-3)\cup(-3,0)$

f decreasing $(0,3)\cup(3,\infty)$

$$f' = \frac{-16x}{(x^2-9)^2}$$

i. Find the second derivative (get a common denominator).

$$f'' = \frac{(x^2-9)^2(-16) - (-16x)\,2(x^2-9)\,2x}{(x^2-9)^4}$$

$$= \frac{\cancel{(x^2-9)}(-16)\left[x^2-9-4x^2\right]}{(x^2-9)^{\cancel{4}\,3}} = \frac{-16(x^2-4x^2-9)}{(x^2-9)^3} = \frac{-16(-3x^2-9)}{(x^2-9)^3} = \boxed{\frac{48(x^2+3)}{(x^2-9)^3}}$$

j. Find all possible points of inflection. (hint: f'' numer = 0 and denom = 0)

$x^2+3=0$ No soln

$x^2-9=0$ $x=\pm 3$ (VA)

No ppoi, No poi

k. Find intervals of concavity. (hint: use a sign line)

f'': $+$ on $(-\infty,-3)$, $-$ on $(-3,3)$, $+$ on $(3,\infty)$; sign line marked at -3 and 3

Concave up $(-\infty,-3)\cup(3,\infty)$

Concave down $(-3,3)$

l. Sketch the graph of $f(x)$. Label intercepts, asymptotes, and max/mins.