AP Calculus AB **Name**_____

Using derivatives to graph a function without a calculator #4

$f(x)=\left(x^2-4\right)^{2/3}$

a. Find the x intercepts. $(2,0)$ $(-2,0)$

b. Find the y intercepts $(0, \sqrt[3]{16})$

c. Find any asymptotes (horizontal and vertical) no VA no HA

d. Find the end behavior. $\lim_{x\to\infty} f(x) = \infty$ $\lim_{x\to-\infty} f(x) = \infty$

e. Find the first derivative (get a common denominator).

$$f'(x)=\frac{4x}{3\sqrt[3]{x^2-4}}$$

f. Find the critical points. (hint: set numerator=0 and denominator = 0) $f'(x)$

$x=0, 2, -2$

g. Use the critical points to find any max/mins. (hint: use a sign line)

local mins $(-2,0)$ $(2,0)$

local max $(0, \sqrt[3]{16})$

h. State intervals of increase and decrease.

increasing $(-2,0) \cup (2,\infty)$

decreasing $(-\infty,-2) \cup (0,2)$

i. Find the second derivative (get a common denominator).

$f' = \dfrac{4x}{3(x^2-4)^{1/3}}$

$f'' = \dfrac{3(x^2-4)^{1/3}\,4 - 4x\cdot\frac{1}{3}\cdot 3(x^2-4)^{-2/3}\,2x}{(3(x^2-4)^{1/3})^2}$

$= \dfrac{12(x^2-4)^{1/3} - \frac{8x^2}{(x^2-4)^{2/3}}}{9(x^2-4)^{2/3}}$

$= \dfrac{12(x^2-4) - 8x^2}{9(x^2-4)^{4/3}}$

$= \boxed{\dfrac{4(x^2-12)}{9(x^2-4)^{4/3}}}$

j. Find all possible points of inflection. (hint: numer = 0 and denom = 0)

$f''(x)$

$x = \pm\sqrt{12} = \pm 2\sqrt{3}$

$x = \pm 2$

f'': + | − | − | − | + at $-2\sqrt{3}$, -2, 2, $2\sqrt{3}$

k. Find intervals of concavity. (hint: use a sign line)

Concave up $(-\infty, -2\sqrt{3}) \cup (2\sqrt{3}, \infty)$

Concave down $(-2\sqrt{3}, -2) \cup (-2, 2) \cup (2, 2\sqrt{3})$

poi $(-2\sqrt{3}, 4)$

$(2\sqrt{3}, 4)$

l. Sketch the graph of $f(x)$. Label intercepts, asymptotes, and max/mins.