

Using derivatives to graph a function without a calculator #4

$$f(x) = (x^2 - 4)^{2/3}$$

- a. Find the x intercepts.  $(2, 0)$   $(-2, 0)$
- b. Find the y intercepts  $(0, \sqrt[3]{16})$
- c. Find any asymptotes (horizontal and vertical) no VA no HA
- d. Find the end behavior.  $\lim_{x \rightarrow \infty} f(x) = \infty$   $\lim_{x \rightarrow -\infty} f(x) = \infty$
- e. Find the first derivative (get a common denominator).

$$f'(x) = \frac{4x}{3\sqrt[3]{x^2-4}}$$

- f. Find the critical points. (hint: set  $f'(x)$  numerator=0 and denominator = 0)

$$x = 0, 2, -2$$

- g. Use the critical points to find any max/mins. (hint: use a sign line)



local mins  $(-2, 0)$   $(2, 0)$

local max  $(0, \sqrt[3]{16})$

- h. State intervals of increase and decrease.

increasing  $(-2, 0) \cup (2, \infty)$

decreasing  $(-\infty, -2) \cup (0, 2)$

i. Find the second derivative (get a common denominator).

$$f' = \frac{4x}{3(x^2-4)^{1/3}}$$

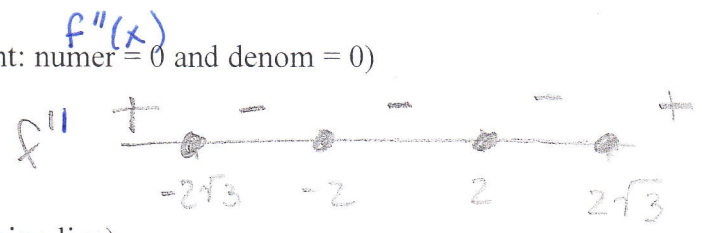
$$f'' = \frac{3(x^2-4)^{1/3} \cdot 4 - 4x \cdot \frac{1}{3} \cdot 3(x^2-4)^{-2/3} \cdot 2x}{(3(x^2-4)^{1/3})^2}$$

$$= \frac{12(x^2-4)^{1/3} - \frac{8x^2}{(x^2-4)^{2/3}}}{9(x^2-4)^{4/3}} = \frac{12(x^2-4) - 8x^2}{9(x^2-4)^{4/3}} = \frac{4(x^2-12)}{9(x^2-4)^{4/3}}$$

j. Find all possible points of inflection. (hint: numer = 0 and denom = 0)

$$x = \pm \sqrt{12} = \pm 2\sqrt{3}$$

$$x = \pm 2$$



k. Find intervals of concavity. (hint: use a sign line)

Concave up  $(-\infty, -2\sqrt{3}) \cup (2\sqrt{3}, \infty)$

Concave down  $(-2\sqrt{3}, -2) \cup (-2, 2) \cup (2, 2\sqrt{3})$

poi  $(-2\sqrt{3}, 4)$   
 $(2\sqrt{3}, 4)$

l. Sketch the graph of  $f(x)$ . Label intercepts, asymptotes, and max/mins.

