AP Calculus

Applications of Derivatives: Extra Practice

1. Find the linear approximation for $h(x) = \sqrt[3]{8-x}$ at $a = 0$ and use it to estimate $\sqrt[3]{7.96}$.

2. A softball diamond is a square whose sides are 60 ft long. Suppose that a player running from first to second base has a speed of 25 $\frac{ft}{\text{sec}}$ at the instant when she is 10 ft from second base. At what rate is the player's distance from home plate changing at that instant?

3. A carpenter is building a rectangular room with a fixed perimeter of 54 ft. What are the dimensions of the largest (max area) room that can be built?

4. A plane flying horizontally at an altitude of 1 mile and a speed of 500 mph passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 miles away from the station.

5. Car A is traveling east at 50 mph away from an intersection. Car B is traveling north at 60 mph towards the same intersection. At what rate are the cars approaching each other when car A is 0.3 miles and car B is 0.4 miles from the intersection?

6. An aircraft has taken off from an airstrip and is climbing at a 30° angle to the horizontal. How fast is the aircraft gaining altitude if its speed is 500 mph?

7. A hobby store has 20 feet of fencing to fence off a rectangular area for an electric train in one corner of its display room. The two sides up against the wall require no fence. What dimensions of the rectangle will maximize the area? What is the maximum area?

8. A ladder 10 ft long rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 1 $\frac{ft}{\sec}$.

a. How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 feet from the wall?

b. Find the rate of change of the angle formed between the ladder and the ground at that same instant.

9. Find the point P on the parabola $y = x^2 - 2$ closest to the point $(0,1)$.

10. A man walks along a straight path at a speed of 4 ft/sec. A searchlight is located on the ground 20 ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 feet from the point on the path closest to the searchlight?

11. A balloon rises at a rate of 3 meters per second from a point on the ground 30 meters from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 30 meters above the ground.

Applications of Derivatives: Extra Practice

(1) $h(x) = \sqrt[3]{8-x} \quad a=0$

$h(0) = 2 \Rightarrow$ pt $(0,2)$ $\qquad h'(x) = \dfrac{-1}{3(8-x)^{2/3}} \qquad h'(0) = \dfrac{-1}{12}$

$$h(x) \approx \frac{-1}{12}(x-0) - 2 = -\frac{x}{12} - 2$$

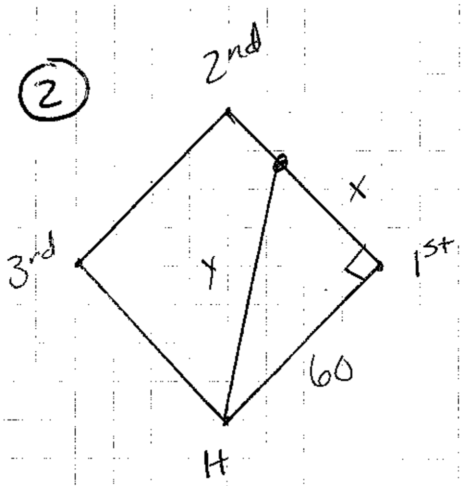
$\sqrt[3]{8-x} = \sqrt[3]{7.96}$

$8-x = 7.96$

$x = 0.04$

$$\sqrt[3]{7.96} \approx \frac{-1}{12}(7.96) - 2 = \frac{-1}{12} \cdot \frac{796}{100} - 2 = \boxed{\frac{-799}{300}}$$

(2)



Given: $\dfrac{dx}{dt} = +25$ ft/sec $\qquad$ Find: $\dfrac{dy}{dt}$

When: $x = 60 - 10 = 50$ ft $\Rightarrow 50^2 + 60^2 = y^2$

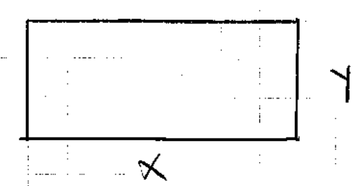
$y = 78.1025$

$x^2 + 60^2 = y^2$

$2x \dfrac{dx}{dt} = 2y \dfrac{dy}{dt}$

$$\frac{dy}{dt} = \frac{x \frac{dx}{dt}}{y} = \frac{(50)(25)}{78.1025} = \boxed{16.005 \ \frac{ft}{sec}}$$

(3)



Optimization Eq

$A = xy$

$A = x(27-x)$

$A = 27x - x^2$

Substitution Eq

$2x + 2y = 54$

$2y = 54 - 2x$

$y = 27 - x$

Optimize: $A' = 27 - 2x = 0$

$x = \dfrac{27}{2}$

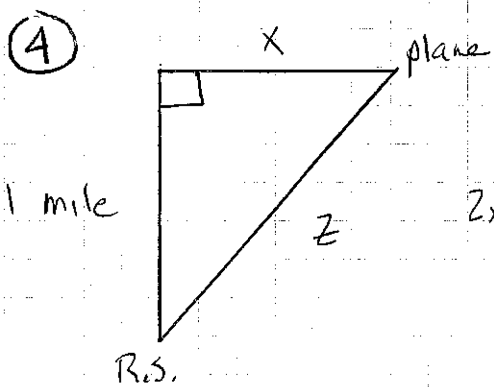
$y = 27 - \dfrac{27}{2} = \dfrac{27}{2}$

Check: $A'' = -2 < 0$

max

Answer: $\boxed{\text{Dims: } \frac{27}{2} \text{ ft} \times \frac{27}{2} \text{ ft}}$

(4)



Given: $\frac{dx}{dt} = 500$ mph Find: $\frac{dz}{dt}$ When: $z=2$

$(x^2+1^2=2^2$
$x=\sqrt{3})$

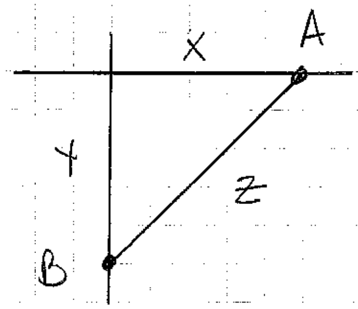
$$x^2+1^2=z^2$$

$$2x\frac{dx}{dt} = 2z\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt}}{z}$$

$$= \frac{(\sqrt{3})(500)}{2} = \boxed{250\sqrt{3}\text{ mph}}$$

(5) Given: $\frac{dx}{dt} = +50$ mph $\frac{dy}{dt} = -60$ mph



Find: $\frac{dz}{dt}$ When: $x=0.3$ $y=0.4$

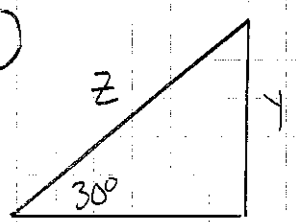
$(z=\sqrt{.3^2+.4^2} = .5)$

$$x^2+y^2=z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{(.3)(50)+(.4)(-60)}{.5} = \boxed{-18\text{ mph}}$$

(6)



Given: $\frac{dz}{dt} = 500$ mph Find $\frac{dy}{dt}$

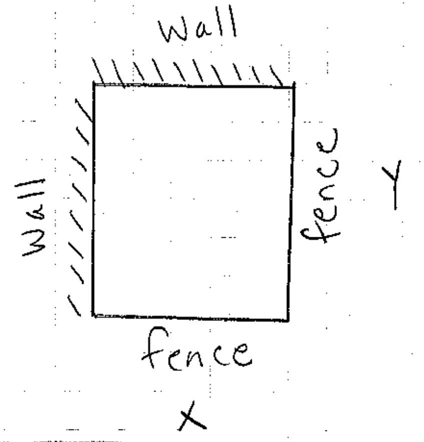
$$\sin 30° = \frac{y}{z} = \frac{1}{2}$$

$$2y = z$$

$$2\frac{dy}{dt} = \frac{dz}{dt}$$

$$\frac{dy}{dt} = \frac{1}{2}\frac{dz}{dt} = \boxed{250\text{ mph}}$$

7



Optimization Eq

$A = xy$

$A = x(20-x)$

$A = 20x - x^2$

Substitution Eq

$x + y = 20$

$y = 20 - x$

Optimize:

$A' = 20 - 2x = 0$

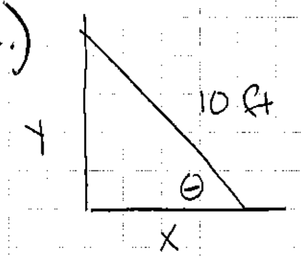
$x = 10$

Check:

$A'' = -2 < 0$
max

$\boxed{\text{Answer: Dims } 10 \text{ ft} \times 10 \text{ ft} \quad \text{Area}_{max} = 100 \text{ ft}^2}$

8 a.)



Given: $\frac{dx}{dt} = +1 \frac{ft}{sec}$ Find: $\frac{dy}{dt}$ When: $x=6$

$(6^2 + y^2 = 10^2$
$y = 8)$

$x^2 + y^2 = 10^2$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

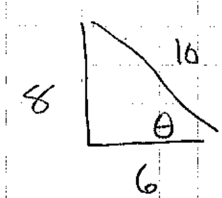
$\frac{dy}{dt} = \frac{-(6)(1)}{8} = \boxed{-\frac{3}{4} \text{ ft/sec}}$

b.) Find $\frac{d\theta}{dt}$

$\sin\theta = \frac{y}{10}$

$\cos\theta \frac{d\theta}{dt} = \frac{1}{10} \frac{dy}{dt}$

$d\theta/dt = \frac{1}{10}\left(-\frac{3}{4}\right) \cdot \frac{5}{3} = \boxed{-\frac{1}{8} \frac{rad}{sec}}$



$\cos\theta = \frac{6}{10} = \frac{3}{5}$

9 $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(x-0)^2 + (y-1)^2}$

$= \sqrt{x^2 + (x^2 - 2 - 1)^2}$

$f(x) = x^2 + (x^2 - 3)^2$

$f'(x) = 2x + 2(x^2 - 3) \cdot 2x = 0$

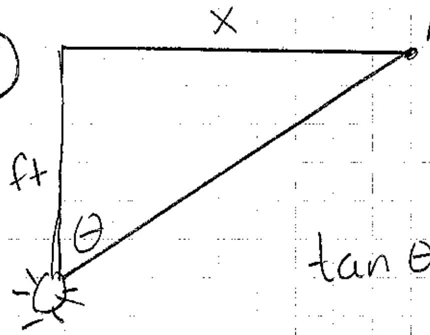
$x = 0, \pm \frac{\sqrt{10}}{2}$

$f''(0) = -10 < 0$ max

$f''\left(\frac{\sqrt{10}}{2}\right) = 20 > 0$ min $\quad f''\left(-\frac{\sqrt{10}}{2}\right) = 20 > 0$ min

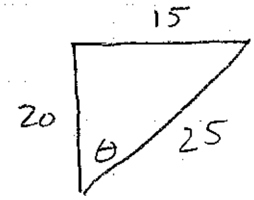
$\boxed{\text{Closest points: } \left(\frac{\sqrt{10}}{2}, \frac{1}{2}\right) \quad \left(-\frac{\sqrt{10}}{2}, \frac{1}{2}\right)}$

(10)



Find: $\frac{d\theta}{dt}$ Given $\frac{dx}{dt} = 4 \ \frac{ft}{sec}$

When $x = 15$ ft



$\tan\theta = \frac{x}{20}$

$\sec^2\theta \ \frac{d\theta}{dt} = \frac{1}{20} \ \frac{dx}{dt}$

$\sec\theta = 5/4$

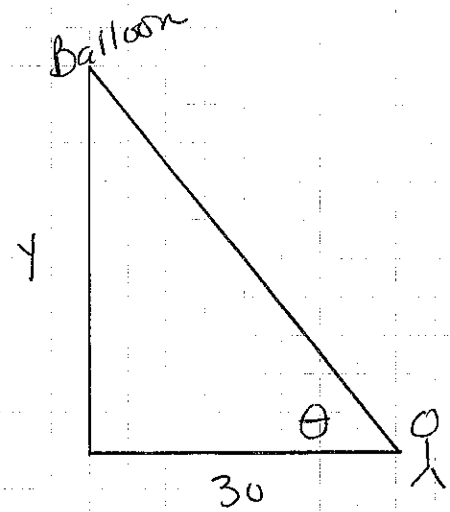
$\sec^2\theta = \frac{25}{16}$

$\frac{d\theta}{dt} = \left(\frac{1}{20}\right)(.4)\left(\frac{16}{25}\right)$

$= \boxed{0.128 \ rad/sec}$

(11) Given $\frac{dy}{dt} = +3 \ m/sec$

Find $\frac{d\theta}{dt}$ When $y = 30$



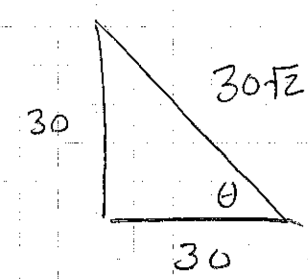
$\tan\theta = \frac{y}{30}$

$\sec^2\theta \ \frac{d\theta}{dt} = \frac{1}{30} \ \frac{dy}{dt}$

$\frac{d\theta}{dt} = \frac{\frac{1}{30}(3)}{2}$

$= \boxed{\frac{1}{20} \ \frac{rad}{sec}}$

When $y = 30$



$\cos\theta = \frac{30}{30\sqrt{2}}$

$\sec\theta = \sqrt{2}$ $\quad \sec^2\theta = 2$